NUMERICAL SOLUTION OF MOVING BOUNDARY PROBLEM RELATED TO CONTINUOUS CASTING

A THESIS SUBMITTED IN PARTIAL FULFILMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology

in

Mechanical Engineering

By

RAJEEV KUMAR

Department of Mechanical Engineering

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Under the Guidance of

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CERTIFICATE

This is to certify that the thesis entitled "**NUMERICAL SOLUTION OF MOVING BOUNDARY PROBLEM RELATED TO CONTINUOUS CASTING**" submitted by **Mr. RAJEEV KUMAR** in partial fulfillment of the requirements for the award of Master of Technology Degree in Mechanical Engineering with specialization in **Thermal Engineering** at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

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ABSTRACT

Metal present in the ore is itself not sufficient for engineering purposes. It should be further refined and some foreign elements are to be added to obtain steel, copper and aluminium metal with different strength, hardness, reliability etc. for different engineering purposes. Competition in metalmaking industry requires the continuously preoccupation with the relevant process and product data for product quality or process productivity assurance and improvement. Recently, energy savings has become the most important theme in the steel manufacturing industry for reasons of environmental protection, economic utilization of resources, reducing capital equipment and reducing transformation cost. To help metalmakers meet ever increasing demands to produce high quality crack-sensitive grades of metal at higher and higher speeds, with enhanced properties and better surface characteristics and slab casting is the process which will help in fulfilling these demands to an extent. In the reported literature, there is a scarcity in the application of numerical analysis on continuous casting due to the involvement of step change in boundary conditions and also due to infinite domain of the medium. In the present study a numerical model, employing the finite difference method, is proposed to solve the conduction equations. The numerical solution of the continuous casting slab problem encounters two major difficulties. The finite difference solution of conduction equation, numerical solution is beneficial in solving conduction equations attributed to temperature dependent thermo physical properties. Finally code was written in FORTRAN-90, and after carring out the simulations the results were analysed.

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NOMENCLATURE

$A_{i,j}$	Coefficients of finite difference equation at nodal location (i,j)
B_i	Biot Number
C	Specific Heat
h	Heat Transfer Coefficient
k	Thermal Conductivity
L	Length of Slab
Pe	Peclet Number
Q	Dimensionless Heat Source parameter
t	Time
T	Temperature
T_c	Coolant Temperature
T_i	Liquid (hot metal) Temperature
T_m	Melting (interface)Temperature
u	Front Velocity
X,Y	Physical Coordinates
$\bar{x},\bar{y}$	Coordinates in quasi-steady state
X,Y	Dimensionless Coordinates in Quasi Steady State

GREEK ALPHABETS

2δ	Thickness of Slab
Γ	Ratio of Solid side to Liquid side Conductivity
θ	Dimensionless Temperature
ρ	Density
α	Thermal diffusivity

SUBSCRIPTS

1	solid Region
2	liquid Region
i ,j	nodal location for finite difference representation

SUPERSCRIPTS

0	nodal location at the center (i, j) of control volume
1, 2, 3, 4	nodal location at (i, j+1), (i+1, j), (i, j-1), (i-1,j)

Chapter-1

Introduction

- General
- History of Continuous Casting
- The basic Principal of Continuous Casting
- Casting Applications
- Closure

CHAPTER-1

INTRODUCTION

1.1 General

Metal present in the ore is itself not sufficient for engineering purposes. It should be further refined and some foreign elements are to be added to obtain steel, copper and aluminium with different strength, hardness, reliability etc. for different engineering purposes. Competition in metalmaking industry requires the continuously preoccupation with the relevant process and product data for product quality or process productivity assurance and improvement. Recently, energy savings has become the most important theme in the steel manufacturing industry for reasons of environmental protection, economic utilization of resources, reducing capital equipment and reducing transformation cost. To help metalmakers meet ever increasing demands to produce high quality crack-sensitive grades of steel at higher and higher speeds, with enhanced properties and better surface characteristics and thin slab casting is the process which will help in fulfilling these demands to an extent. Heat transfer in the continuous slab casting mold is governed by many complex phenomena.

Figure 1 shows a schematic of some of these. Liquid metal flows into the mold cavity through a submerged entry nozzle, The direction of the steel jet controls turbulent fluid flow in liquid cavity, which affects delivery of superheat to solid/liquid interface of the growing shell. The liquid steel solidifies against the four walls of the water-cooled copper mold, while it is continuously withdrawn downward at the casting speed.

Mold powder added to the free surface of the liquid steel melts and flows between the steel shell and the mold wall to act as a lubricant so long as it remains liquid. The resolidified mold powder, or “slag”, adjacent to the mold wall cools and greatly increases in viscosity, thus acting like a solid. It is thicker near and just above the meniscus, where it is called the “slag rim This relatively solid slag layer often remains stuck to the mold wall, although it is sometimes dragged intermittently downward at an average speed less than the casting speed. This thesis first describes the formulation of this model, which has been implemented into user-friendly FORTRAN program workstations.

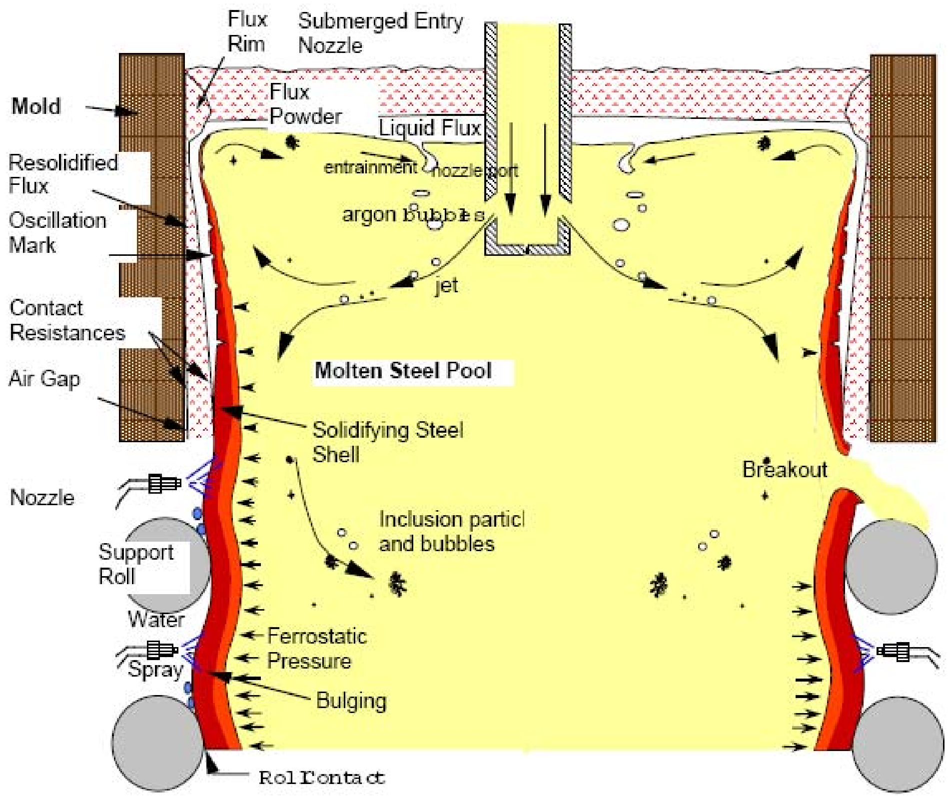


Figure 1.1 Schematic of continuous casting process showing slag layers

1.2 History of continuous casting

After the iron is extracted from its ore, it is taken to LD furnace where proper percentage foreign elements are added to get the steel of required properties. In early 1950’s L.D. Process was developed by Liz of Germany and Donwitz of Austria to make steel. The oxygen is blown in the L.D. converter from bottom to convert molten iron into steel by removing its carbon, silicon, sulfur and phosphorous contents. The liquid steel from the converter is converted to slabs using Continuous Casting machine. There are two methods of casting steel slabs – horizontal continuous casting method and thin slab casting method**.**

In thin slab casting we don’t require heating before hot rolling as in conventional continuous casting. The cooling rate of strip in thin slab casting is high. The thin slab caster eliminates the need for a roughing mill in the hot-rolling process. Moreover it is found that the inclusions have hardly any effects on the microstructures and tensile properties of the strip even there is a refining tendency for the inclusions in thin slab casting. Thin slab casting and rolling is the lowest cost method for hot-band production and economically satisfy the engineering need with less steel and thus it lowers the power consumption on the processes from which it goes through. Thin slab continuous-casting machines produce a slab approximately 50-60 mm thick. This significantly reduces the amount of hot rolling required to produce thin sheet, thus allowing for in-line hot rolling of steel as it comes off the caster. However, because the slab produced by thin slab casting machines is 1/5 the thickness of that produced by conventional thick slab casting, the thin slab caster cast approximately five times faster to match the productivity of the conventional caster. In view of the increasing generation of scrap in the industrialized countries, the scrap-EAF-thin slab caster combination is gaining an increasing share in world steel production. Thin slab process is becoming more and more attractive in the steel market also for the most demanding applications, once approachable from thick slab process only.

Hot Liquid Iron is converted to Steel in the Steel Melting Shops. Hot Metal from the Blast Furnace is stored in Mixers in LD vessel. The Hot Metal is converted to Steel in the LD converters by removing its carbon, silicon, sulphur and phosphorous contents. The liquid steel from the converter is converted to slabs using Continuous Casting machine. The liquid steel is treated in on line purging, Ladle Refining Furnace or Argon Rinsing station before continuous casting. The Steel Melting Shop requires an Oxygen Plant to cater to the requirement of oxygen for steel making.

1.3 THE BASIC PRINCIPLES OF CONTINUOUS CASTING

The Process

Continuous casting is the process whereby molten metal is solidified into a "semi finished" billet, bloom, slab or beam blank. Prior to the introduction of continuous casting in the 1950s, steel was poured into stationary moulds to form "ingots". Since then, "continuous casting" has evolved to achieve improved yield, quality, productivity and cost efficiency. Nowadays, continuous casting is the predominant way by which steel is produced in the world. Continuous casting is used to solidify most of the 750 million tons of steel, 20 million tons of aluminum, and many tons of other alloys produced in the world every year.

In the continuous casting process, illustrated in Figure 1, molten metal is poured from the ladle into the tundish and then through a submerged entry nozzle into a mould cavity. The mould is water-cooled so that enough heat is extracted to solidify a shell of sufficient thickness. The shell is withdrawn from the bottom of the mould at a "casting speed" that matches the inflow of metal, so that the process ideally operates at steady state. Below the mould, water is sprayed to further extract heat from the strand surface, and the strand eventually becomes fully solid when it reaches the "metallurgical length".

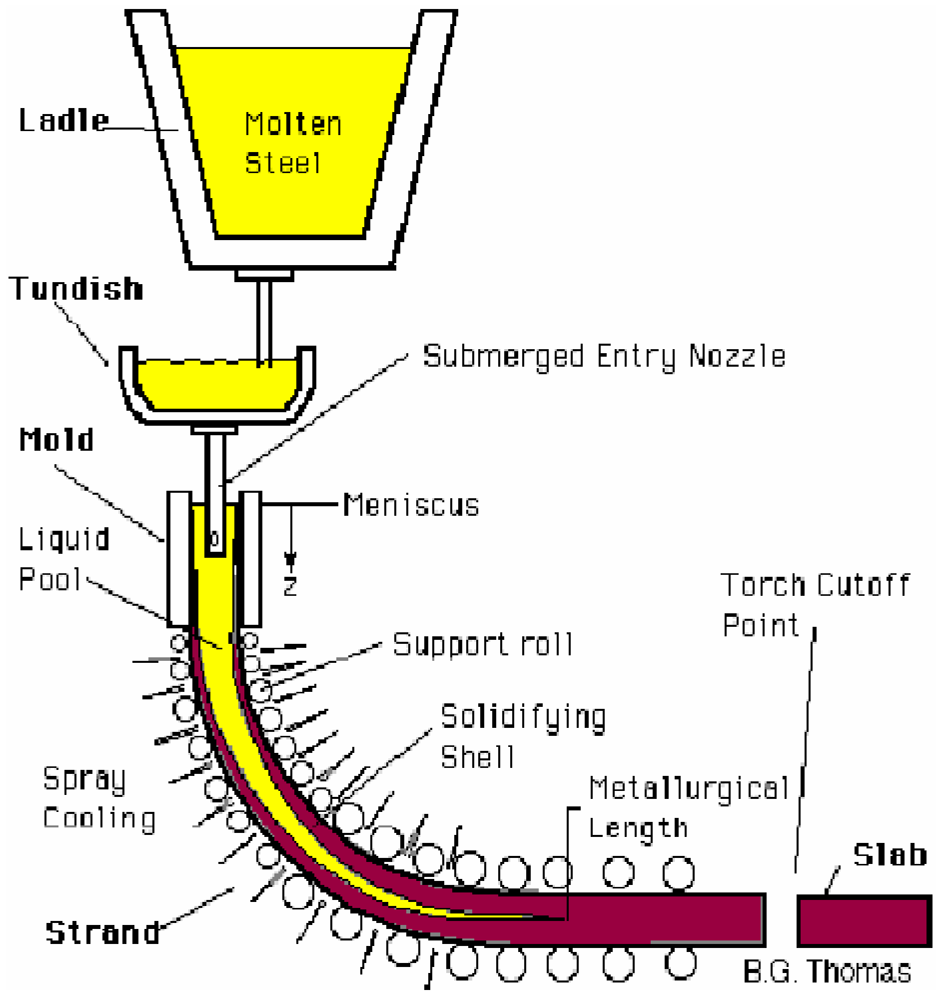


Figure 1.2 Schematic representation of the continuous casting process

Solidification begins in the mould, and continues through the different zones of cooling while the strand is continuously withdrawn at the casting speed. Finally, the solidified strand is straightened, cut, and then discharged for intermediate storage or hot charged for finished rolling.

To start a cast, the bottom of the mould is sealed by a steel dummy bar. This bar prevents liquid metal from flowing out of the mould and the solidifying shell until a fully solidified strand section is obtained. The liquid poured into the mould is partially' solidified in the mould, producing a strand with a solid outer shell and a liquid core. In this primary cooling area, once the steel shell has a sufficient thickness, the partially solidified strand will be withdrawn out of the mould along with the dummy bar at the casting speed. Liquid metal continues to pour into the mould to replenish the withdrawn metal at an equal rate. Upon exiting the mould, the strand enters a roller containment section and secondary cooling chamber in which the solidifying strand is sprayed with water, or a combination of water and air (referred to as "air-mist") to promote solidification. Once the strand is fully solidified and has passed through the straightener, the dummy bar is disconnected, removed and stored.

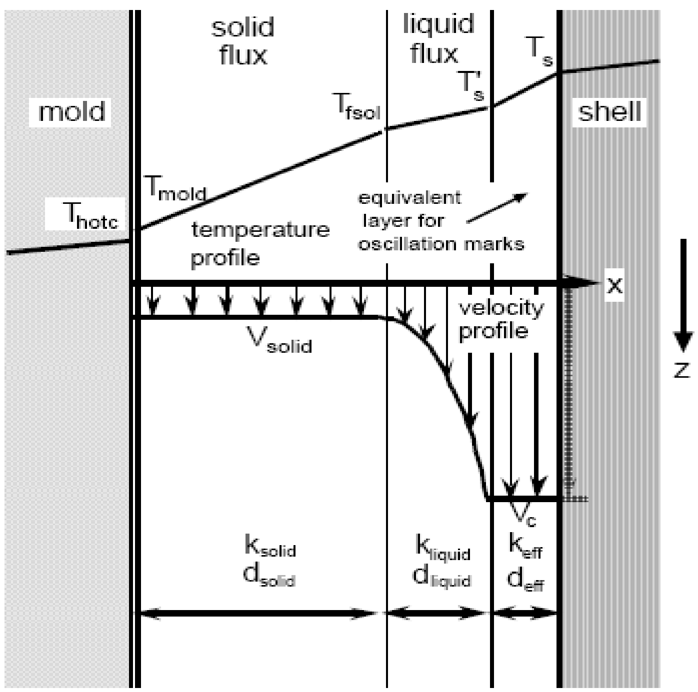


Figure 1.3 Velocity and temperature profiles assumed across interfacial gap

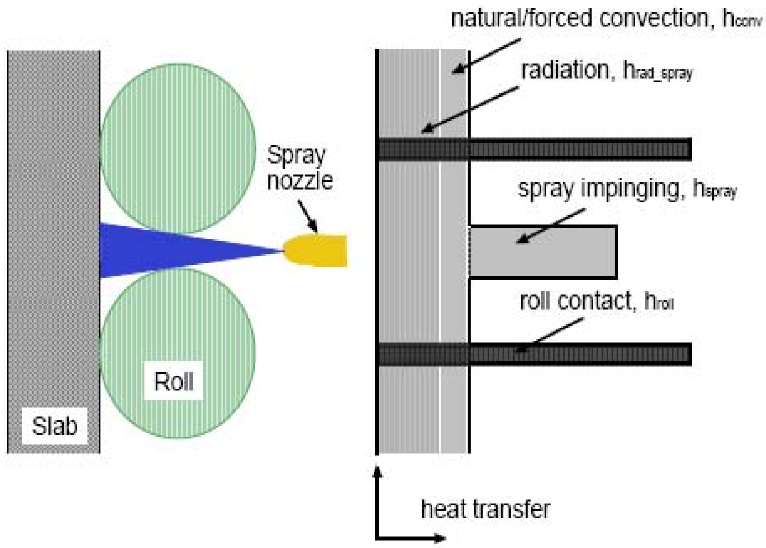


Figure 1.4 Schematic of spray zone region

This analysis starts with some of the ideas in the second analytical method of and provides a procedure to deal with further generalized boundary conditions for ingot casting figure shows the ingot being cast at velocity u.

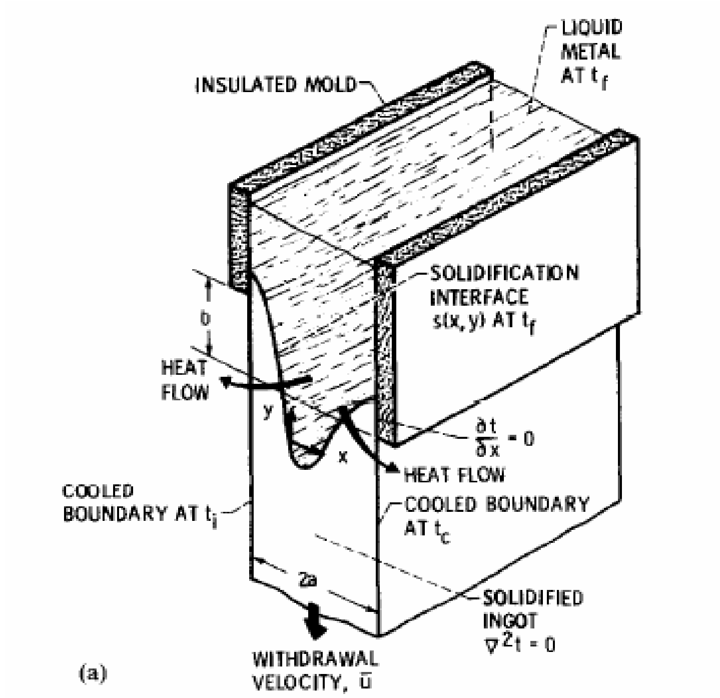


Fig 1.5 Slab ingots with unequally cooled sides being withdrawn from offset mold in continuous casting. Physical conditions.

1.4 casting applications

Moving the interface according to casting speed

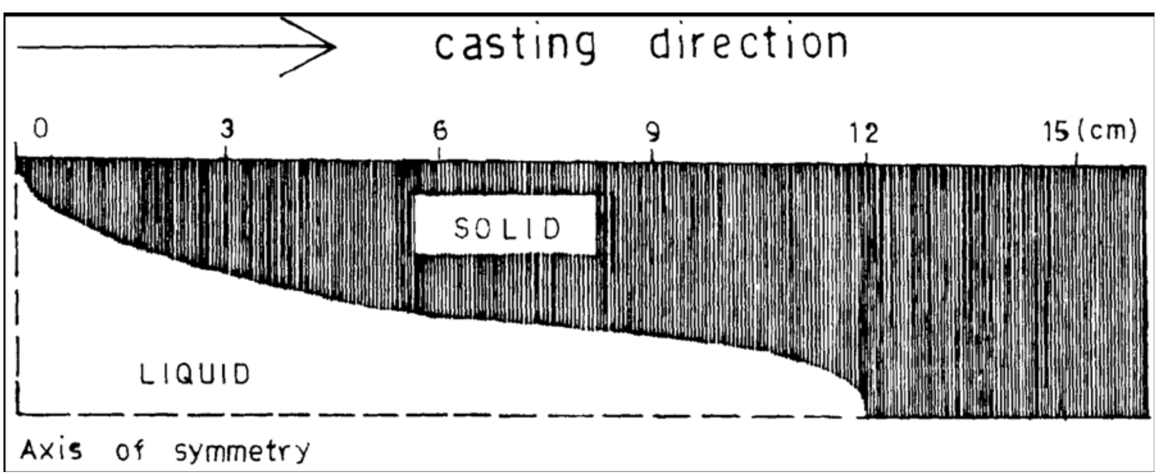


Figure 1.6 Solidification profile (casting speed = 0·0015 m/sec and lateral dimension of the strand= 0·17 m).

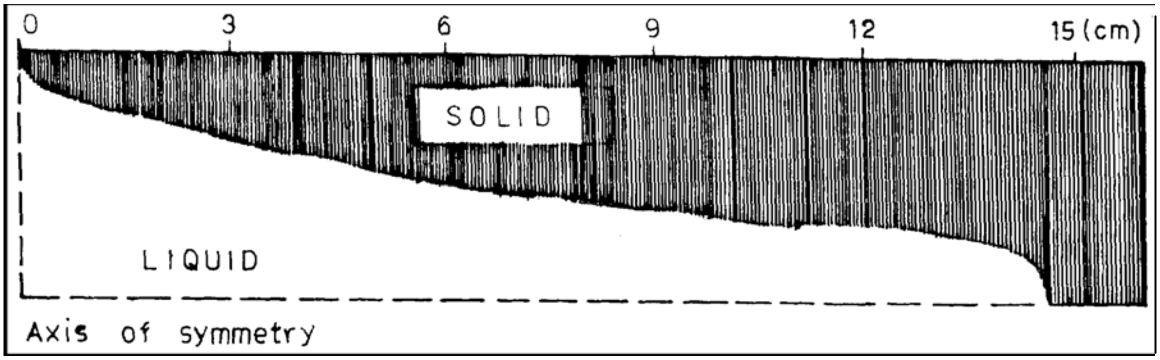


Figure 1.7 Solidification profile (casting speed = 0.002m/sec and lateral dimension of the strand = 0.17m)

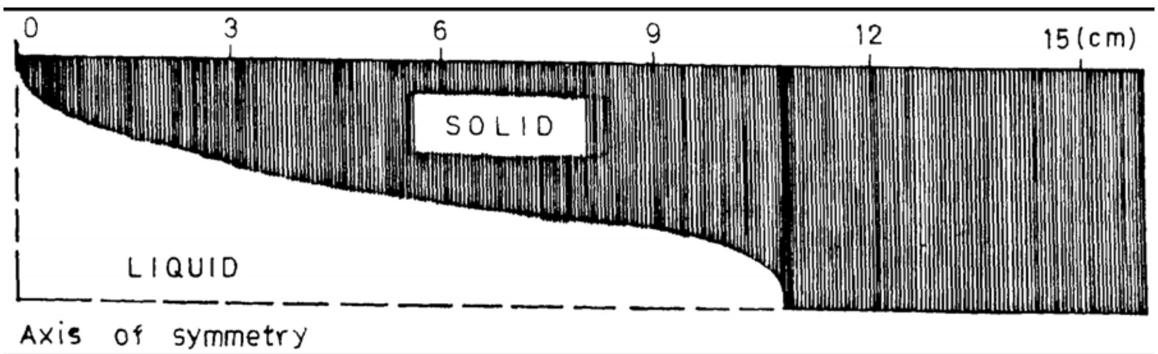


Figure 1.8 Solidification profile (casting speed =0.00175m/sec and lateral dimension of the strand = 0.17m)

Three different casting speeds have been considered in the analysis. These speeds are 0·0015 m/sec,0·002 m/sec and 0·00175 m/sec. The number of collocation points in *x*-direction, $n = 22$. The casting condition and operating parameters are identical in all these three cases except the casting speed. The interface profile evaluated on the basis of present approach is found to be in good agreement with other numerical investigations by different formalisms (Udaykumar 1993; Shyy 1993). Although, a point to point spatial mapping of the interface profile may not be realizable, but the general trend is quite satisfactory. This is quite evident from the figures 1–3 and it is self-explanatory.

1.5 Closure

Heat conduction controlled continuous casting models are usually used to explain and predict the slab characteristics. Several analytical and numerical solutions exist for single geometries. From the numerical point of view, the solution of the continuous casting problem is rather difficult. One would like to have a method whose accuracy is independent of Peclet numbers. However, larger the Biot number is, the steeper is the gradient of temperature profile at the slab interface. This, together with the infinite domain, makes the use of finite differences method.

Chapter-2

Literature Review

- **Introduction**
- **Numerical Solutions**
- **Analytical Solutions**
- **Experimental Investigation**

CHAPTER – 2

2. LITERATURE SURVEY

2.1 Introduction

The literature reviewed in this chapter can be broadly classified under three categories. The first part of the survey deals with the numerical solutions for conduction equation in continuous casting as described in chapter 2.2. This contains the solution for one and two-dimensional models, slab for two region heat transfer and also the rewetting models with boundary heat flux. The various numerical techniques adopted in the literature are finite difference method, finite element method, and implicit isotherm migration technique.

The second part of the survey deals with analytical solutions for quasi-steady heat conduction equation for continuous casting slab, as described in chapter 2.3. The third part of the survey deals with experimental investigations in various geometries in vertical or horizontal configurations, as described in 2.4. The main objectives of the experimental studies include measurement of casting withdrawal velocity under different casting speed conditions.

2.2 Numerical Solutions

Solidification process proceeding in the domain casting mould is described by the system of partial differential equations and boundary/initial conditions. Majchrzaka Ewa. Mochnackib Bohdan, Szopa Romuald [1] applied the combined variant of the boundary element method called the BEM using discretization in time. In particular the well-known numerical algorithm for domains oriented in the Cartesian co-ordinate system was considered. In order to adapt the method for numerical simulation of the heat-transfer processes proceeding in domains oriented in other co-ordinate systems a certain algorithm consisting in the introduction of an artificial source was used. So the concept of an artificial source introduction for the numerical solution of thermal diffusion problems in cylindrical or spherical domains can be treated as a quite good method generalizing the range of the discussed variant of the BEM applications.

Fairuzov. Y. V [2] examined the transient conjugate two-phase heat transfer during depressurization of pipelines containing flashing liquids. A numerical model for transient flashing liquid flow in a pipe was formulated. The model takes into account the transient radial heat conduction and the forced convection effects. Numerical simulation of flashing two component (propane and butane) flows was performed in order to investigate the effect of wall friction on the heat transfer conditions in the pipe. The simulation results were compared with predictions of the model that are based on a new formulation of energy equation proposed by the author in an early study. The results helped in order to determine the range of applicability of the new energy equation formulation. A procedure was proposed for choosing an appropriate model for predicting transient conjugate two-phase heat transfer during releases of flashing liquids from pipes. A criterion of thermodynamic similarity for flashing liquids flows in pipes or channels was formulated. The proposed criterion provided the basis for selecting model fluids and for constructing experimental models of systems containing flashing (volatile) liquids with scaled thermodynamic conditions.

Fikiin A.K [3] has covered a great variety of unsteady heat conduction cases accompanied by phase transformations. A mathematical model was developed for determination of the unsteady-state temperature and enthalpy fields (as well as the space time evolution of the phase content) and of the cooling and freezing (heating and thawing) times of food materials and other bodies of various configuration (representing multicomponent two-phase systems having one freezable component). The author proposed an improved enthalpy method by which all non linearities, caused by the temperature dependence of the thermo physical coefficients, are introduced in a functional relationship between the volumetric specific enthalpy and the Kirchhoff function. Thus the non-linearity can be eliminated as a factor making the solution difficult. The applied approach possesses great adaptivity and flexibility in solving complicated moving boundary problems: it is suitable for both isothermal and non-isothermal phase change, reaches a high degree of correspondence between the real physical phenomenon and **its** mathematical formalization, uses uniform and easy fixed-grid computational techniques, makes it possible to avoid complications and to eliminate possible errors caused by 'jumping' of the equivalent specific heat capacity peak at the maximum of the latent heat effect, etc. Efficient procedures and algorithms for computer simulation of complex refrigerating technological processes were created.

Hsieh.C.K [4] referred the source-and-sink method to be attractive in the solution of the Stefan problems. Using one set of equations to solve temperatures in four regions, the source-and-sink method can be ideally suited to the solution of the present problems by Laplace transform. In this method, the general solutions are expressed in terms of the temperature and its slopes at both sides of the moving front. The boundary conditions can thus be applied readily to complete the solution.

Seshadri.R and Krishnayya .A.V.G [5] has determined the thaw or frost depth below heated or chilled insulated structures. Specifically, the method is applied to buried circular pipes, infinite strips and circular discs. For the case when the ground temperature is different from the phase change temperature, the solution is obtained by numerical integration of a quadrature. For the case when the ground temperature approaches the phase change temperature closed form solutions are obtained. The quasisteady analysis is to determine the movement with respect to time of the interface between the unfrozen and frozen regions below the ground surface, for specific geometries such as buried pipe, strip footing and tanks with circular base. Closed form solutions are obtained, when the ground temperature approaches the phase change temperature. When the ground temperature is different from phase change temperature. The solutions are obtained using numerical integration.

ROBERT &EGEL [6] have made an analysis of the two-dimensional, interface shape of a slab ingot being cast continuously by withdrawing it from a mold. The sides, q & the ingot are being cooled and the upper boundary of the ingot is in contact with a pool of molten metal. The solidification interface shape was determined from the analysis of the heat flow, utilizing the condition that the solidification interface is at constant temperature and must be normal to the lines of heat flow carrying away latent heat of fusion from the interface. The shape was found to depend on only one dimensionless parameter that involves the casting rate, the width of the ingot, and the cooled boundary temperature imposed by the coolant. This parameter governs the curvature of the solidification interface and thus shows what conditions must be imposed to achieve a desired flatness of the interface; the flatness has an influence on the microscopic crystal growth at the interface. The thickness of the solidified material was found to increase approximately as the square root of the distance along the mold from the location where solidification begins.

R, S. Gupta and Dhirendra Kumar [7] have obtained the numerical results for solidification of a liquid initially at its fusion temperature.

Hartnett.J.P and Minkowycz [9] .W.J have derived the exact analytical solution for the unidirectional transient (unsteady) heat or mass transfer problems with an axially moving boundary in semi-infinite domains. Then, a comparison between the transient and quasi-steady state solutions is provided for the temperature (or concentration) distribution, temperature (or concentration) gradient at the moving boundary, and thermal (or solutal) boundary layer thickness. They demonstrated that large differences exist between the transient and quasi-steady state solutions for heat or mass transfer problems with an axially moving boundary. The quasi-steady state approach can provide accurate results (less than 1% solution error as compared with the exact transient solution). It has also been shown that the quasi-steady state approach well-under predicts the solid/liquid interface temperature and concentration gradients as well as the temperature and concentration distributions and well-over predicts the thickness of both the thermal and solutal boundary layers.

Shankar Krishnan and Jayathi Y. Murthy [11] have investigated the determination for Transient solid-liquid phase change occurring in a phase-change material (PCM) embedded in metal foam. Natural convection in the melt was considered. Volume-averaged mass and momentum equations are employed, with the Brinkman- For chheimer extension to the Darcy law to model the porous resistance. Owing to the difference in the thermal diffusivities between the metal foam and the PCM, local thermal equilibrium between the two is not assured. Assuming equilibrium melting at the pore scale, separate volume-averaged energy equations are written for the solid metal foam and the PCM and are closed using an interstitial heat transfer coefficient. The enthalpy method is employed to account for phase change. The governing equations are solved implicitly using the finite volume method on a fixed grid. The influence of Rayleigh, Stefan, and interstitial Nusselt numbers on the temporal evolution of the melt front location, wall Nusselt number, temperature differentials between the solid and fluid, and the melting rate was documented. In many applications, periodic pulsed heating may be used. The difference in response time between systems with and without metal foam enhancers has important implications for the management of transient energy pulses. If the time scale of the energy pulse is short compared to the response time of the system, local overheating is possible. Since the metal foam response time is typically far faster than typical energy pulse time scales, it would tend to perform far better than systems without metal foams.

LAITINEN .E and NEITTAANMAKI .P [12] have simulated a steady-state nonlinear parabolic-type model, which is the multiphase heat transfer during solidification in continuous casting. An enthalpy formulation was used and a FE-method in space and an implicit Euler method in time were applied. We compute the temperature distributions in the strand when the boundary conditions (mold/spray cooling) on the strand surface are known. The numerical model gives thereby a good basis for the testing of new designs of continuous casting machines.

AKS [13] has presented the finite-element solution of two-dimensional convection {diffusion equation in an infinite domain, arising out of quenching of an infinite slab. The solution gives the quench front temperature as a function of various model parameters, such as Biot number and Peclet number. The results show good agreement with available closed-form solutions, thus validates the numerical procedure adopted. It is therefore expected that the present method of solution may be extended to quenching problems involving heat generation and precursory cooling, etc., in various other geometries. A numerical solution for solving infinite domain problems arising out of rewetting analysis has been suggested. The infinite physical domain can be mapped to a infinite computational domain by transforming the governing equation. The value of the stretching parameter used for infinite transformation can be obtained by minimizing the heat balance. Quench front temperature is observed to increase with increase in Peclet number and with decrease in Biot number. It is felt that the present solution procedure, in principle, may be extended to other infinite domain rewetting problems in various other geometries.

An analytical solution for rewetting of an infinite slab with a uniform heating has been obtained, employing the Wiener-Hopf technique. In general, quench front temperature is found to increase with increase in Peclet number and dimensionless heat flux, and with decrease in Biot number. The boundary conditions in the present formulation require liquid/vapor temperatures and liquid/ vapor heat transfer coefficients as input parameters, these limitations being inherent in a conduction-controlled rewetting model. The arbitrariness of the choice of their values may be eliminated if a conjugate heat transfer model is considered, where the energy equations of solid, liquid and vapor regions need to be solved simultaneously.

A numerical study has been made to investigate the effect of internal heating and precursory cooling during quenching of an infinite tube. The finite difference solution gives the quench front temperature as a function of various model parameters such as Peclet number, Biot number and dimensionless heat flux. The parametric dependence of the rewetting rate is obtained by the condition that the surface can only be wetted when its temperature is below the quench front temperature. Also, the critical heat flux is obtained by setting Peclet number equal to zero, which gives the minimum heat flux required to prevent the hot surface being rewetted. The numerical model is validated by comparing the results with known closed form solutions

Lazaridis Anastas [14] developed a simple numerical technique with which to treat heat-transfer problems involving a change of phase. These problems are nonlinear due to the conditions at the moving interface boundary surface. The numerical scheme presented here solves the pertinent equations for the multidimensional problem and determines the temperature distribution in both media around the liquid-solid interface while at the same time it locates the loci of the latter as time progresses. The types of boundary conditions most frequently encountered in practice are studied in the analysis; the sample problems are selected in such a way as to reflect constant temperature and Newtonian cooling conditions at the boundary of the solidifying substance. The two-dimensional slab and the two- and three dimensional comers are used to exemplify typical multidimensional geometries. Comparisons of the results obtained in this work with the few existing solutions show satisfactory agreement

Voller V. R. and Prakash. C [15] developed an enthalpy formulation based fixed grid methodology is for the numerical solution of convection-diffusion controlled mushy region phase-change problems. The basic feature of the proposed method lies in the representation of the latent heat of evolution, and of the flow in the solid-liquid mushy one, by suitably chosen sources. There is complete freedom within the methodology for the definition of such sources so that a variety of phase-change situations can be modeled. A test problem of freezing in a thermal cavity under natural convection is used to demonstrate an application of the method. They developed a generalized methodology for the modeling of mushy region phase change. This motivated the development of a fixed grid approach along with retaining the basic form of the hydro mechanical equations. The phenomena associated with a particular phase change can be modelled on careful consideration and choice of source terms. The driving source terms are the 'Dar& source and the latent heat source. The Darcy source is used to model the effect of the nature of the porosity of the mushy region on the flow field. Preliminary results suggest that the nature of the porosity has a significant effect. The latent heat source term is a function of the solid fraction which is a function of temperature. In this paper a linear change was assumed. In real systems the solid fraction-temperature relationship may not be such a simple form. In a binary alloy for example ***F,*** will depend on the nature of the solitte redistribution and may take a non-linear form possibly with a jump discontinuity at a eutectic front. There is a need for further studies to be made. A comparison between the proposed fixed grid method and a deforming grid technique. Such a study would provide a mechanism by which the relative advantages and disadvantages of each approach could be analyzed. Odotogy to metal systems, where the flow in the mushy zone is significant.

Voller.V and Cross .M [16] have described the conventional numerical implementations of Stefan problems using the enthalpy formulation, a simple development which leads to very accurate solutions. The extension of this technique to two dimensional problems is then demonstrated using a straightforward explicit method. An implicit scheme for one dimensional problems, based upon the above development, is then described which can cope with any size phase change temperature range and the influence of internal heating, simultaneously. Finally, the utility of this scheme is demonstrated by its application to a welding problem. They described the explicit finite difference solution to the enthalpy formulation which provides accurate solutions to Stefan problems, whether the phase change occurs at a specific temperature or across a range. The non physical features usually associated with enthalpy methods are eliminated, the resulting algorithm is extremely simple to implement and solutions with a relative accuracy of 0.1% have been obtained. The explicit algorithm has been extended to two dimensional regions and a problem solved which produces stability. The principle used in the simple one-dimensional method has also been exploited to develop an implicit algorithm which is both accurate and fast. This implicit method is capabIe of producing stable solutions to problems where the phase change region varies from a point to a temperature range during its solution and internal heating both occur simultaneously. Finally, by carefully reinterpreting the numerical results from the standard enthalpy method powerful numerical tools have been developed to solve complex Stefan problems. The main advantages of the modified explicit enthalpy method and the implicit “node jumping” scheme may be summarized as : (i) simple in concept and easy to program, (ii) no starting solution required, (iii) accurately tracks the phase change boundary and the temperature history curves at any point, (iv) copes easily with non-constant thermal properties, (v) deals with problems involving any size phase change temperature range (including a single point) and body heating, simultaneously, and (vi) extends easily to multi-dimensional problems.

Das S K [18] has described a cubic spline based collocation method to determine the solid–liquid interface profile (solidification front) during continuous casting process. The basis function chosen for the collocation algorithm to be employed in this formalism, is a cubic spline interpolation function. An iterative solution methodology has been developed to track the interface profile for copper strand of rectangular transverse section for different casting speeds. It is based on enthalpy conservation criteria at the solidification interface and the trend is found to be in good agreement with the available information in the literature although a point to point mapping of the profile is not practically realizable. The spline based collocation algorithm is found to be a reasonably efficient tool for solidification front tracking process, as a good spatial derivative approximation can be achieved incorporating simple modelling philosophy which is numerically robust and computationally cost effective. He found to be an efficient method for tracking the solid–liquid interface profile during continuous casting of metals. In this study, an effort has been made to track the solid liquid interface i.e. the solidification front by developing effective spline based collocation formalism. The formulation has been applied to the continuous casting of copper strand for different casting speeds under a given operating condition. It is found to be in good agreement with other published investigations with regard to the tracking of solidification interface profiles for metals and alloys which testifies that the algorithmic approach is correct in its entirety.

Fairuzov Y. V [19] examined the transient conjugate two-phase heat transfer during depressurization of pipeline examined containing flashing liquids. A numerical model for transient flashing liquid flow in a pipe is formulated. The model takes into account the transient radial heat conduction and the forced convection effects. Numerical simulation of flashing two component (propane and butane) flows is performed in order to investigate the effect of wall friction on the heat transfer conditions in the pipe. The simulation results are compared with predictions of the model that are based on a new formulation of energy equation proposed by the author in an early study. A comparison of the results obtained allows one to determine the range of applicability of the new energy equation formulation. A procedure is proposed for choosing an appropriate model for predicting transient conjugate two-phase heat transfer during releases of flashing liquids from pipes. A criterion of thermodynamic similarity for flashing liquids flows in pipes or channels is formulated .The proposed criterion provides the basis for selecting model fluids and for constructing experimental models of systems containing flashing (volatile) liquids with scaled thermodynamic conditions. The problem of transient, two-phase, conjugate heat transfer during the depressurization of pipelines conveying flashing liquids is examined in this study. A numerical model for transient flashing liquid flow has been developed. The model rigorously takes into account the transient radial heat conduction and the forced convection effects. The model has been validated using previously published experimental data. The effect of friction on the heat transfer conditions in pipes has been studied. Numerical simulation of flashing two-component liquid flow was performed to determine the range of applicability of the solution based on the new formulation of energy equation. Relying on the simulation results a procedure was proposed for choosing an appropriate model for predicting transient conjugate two-phase heat transfer during release of flashing liquids from pipes. A criterion of thermodynamic similarity for flashing liquid flows in pipes or channels has been formulated. The proposed condition provides the basis for selecting model fluids, as well as for constructing experimental models of systems containing flashing ~volatile! liquids with scaled thermodynamic conditions.

Barbosa.Jader R and Hewitt .Geoffrey. F [20] presented a calculation methodology to predict the peaks in heat transform coefficient at near zero equilibrium quality observed in forced convective boiling in vertical conduits. The occurrence of such peaks is typical of low latent heat, low thermal conductivity systems (such as refrigerants and hydrocarbons), and of systems in which the vapor volume formation rate for a given heat flux is large (low-pressure water). The methodology is based on a model that postulates that the mechanism behind the heat transfer coefficient enhancement is the existence of thermodynamic non equilibrium slug flow, i.e., a type of slug flow in which rapid bubble growth in sub cooled boiling leads to the formation of Taylor bubbles separated by slugs of sub cooled liquid. Results are compared with experimental data for forced convective boiling of pure hydrocarbons and show considerable improvement over existing correlations. They presented a model for predicting the heat transfer coefficient peaks observed in the near zero quality region in boiling of hydrocarbons in a vertical pipe associated with the formation of a type of slug flow in which the Taylor bubbles are separated by sub cooled liquid slugs. The main conclusions arising from this work are as follows: It was shown that the heat transfer coefficient peaks coincide with the peaks in the calculated difference between the equilibrium bulk and average slug temperatures, remain sub cooled for distances longer than would be the case for equilibrium flow situations. A consequence of this effect is that the wall temperature in the liquid slug region is lower than that in the equilibrium case.

Satapathy .A. K and Sahoo.R.K determined the two-dimensional quasi-steady conduction equation governing conduction controlled rewetting of an infinite slab, with one side flooded and the other side subjected to a constant heat flux, has been solved by Wiener-Hopf technique. The solution yields the quench front temperature as a function of various model parameters such as Peclet number, Biot number and dimensionless heat flux. Also, the critical (dry out) heat flux is obtained by setting the Peclet number equal to zero, which gives the minimum heat flux required to prevent the hot surface being rewetted.

2.3 Analytical solutions

BISCHOFF.K.B [8] has concluded that the problem of heterogeneous solid-fluid chemical reactions with a moving boundary is commonly treated by means of the pseudo steady state approximation. It was found that for solid-gas systems the pseudo steady state approximation is valid but for solid-liquid systems it may be in error.

Lin Sui and Jiang Zheng [17] investigated by using the improved quasi-steady analysis model developed in the present study. In the improved quasi-steady analysis, an additional term is added to the temperature profile to simulate the transient effect on the temperature distribution in the solid phase. This additional term is based on the ratio of the heat flux at the phase boundary to that at the cooling surface, and physically, presents the thermal capacity effect in the frozen region. The maximum relative error of the moving phase front location obtained from the improved quasi-steady analysis is about 3% in comparison with that obtained from the exact solution of the freezing process in a plate. Since there is no exact solution available for the freezing process taking place in a cylinder or a sphere, the results obtained from the improved quasi-steady analysis are compared with results from references. The maximum relative errors of the improved quasi-steady analysis for the cylindrical and spherical cases are less than 4% while the maximum relative errors of the quasi-steady approximation are higher than 42%. It is evident that the improved quasi-steady analysis developed in the present study maintains the simplicity of the quasi-steady approximation while greatly increasing its accuracy. For the quasi-steady approximation, when the thermal capacity in the frozen solid is neglected, the temperature gradient at the solidification front is larger than that of the reality. As a result, the velocity of the moving solidification front is higher than the reality because this velocity is proportional to the temperature gradient at the solidification .Improved quasi-steady analysis developed in the present study maintains the simplicity of the quasi-steady approximation while greatly increasing its accuracy.

Siegel Robert [21] during solidification, have investigated the shape of the solid-liquid interface is important as it influences the resulting crystal structure. In continuous casting, where an ingot is being withdrawn from a mold, the solidification interface (which is a free' boundary) is regulated by the cooling conditions and mold shape. Two analytical methods were given that yielded exact solutions for the free-boundary shapes. It was shown that it is much more convenient to obtain results by an inverse-type of solution wherein the physical coordinates are dependent variables of orthogonal temperature and heat flow functions. This type of solution will be further developed here to obtain solidification-interface shapes for more complex situations wherein both the ingot cooling and mold Greek symbols geometry are asymmetric.

2.4 Experimental Investigations

Kazuo Takeda a and Yoshisuke Nakano b[10] have conducted a series of freezing tests on three kinds of soil to find the steady growth condition of a segregated ice layer by using a new steady-state method in which the temperature profiles of soil specimens were controlled. It was found that the steady growth condition is determined by the absolute value of the temperature gradient of the unfrozen part of the soil au and that of the frozen part of the soil and Comparing these experimental results with the results of the mathematical analysis.

Chapter-3

Mathematical Formulation

- Introduction
- Governing Equation and Boundary Conditions
- Governing Equations
- Finite –Difference Equations
- Closure

CHAPTER – 3

Finite Difference Solution of Slab

3.1 Introduction

In variety, of scientific and engineering applications, one often needs to solve partial differential equations in unbounded domains. For the standard numerical methods, such as finite difference and finite element methods, it is difficult to establish the conventional techniques of dicretisation of the domain for solving boundary value problems for an infinite domain.

In view of engineering treatment, it is quite common to replace the original infinite domain problem by one in a finite domain which is considered to be “sufficiently large” and, therefore the whole computation process will be time consuming. Many methods have been developed to solve the above problems accurately. In one such method, known as the domain transformation method ,the infinite physical domain can be reduced to a finite computational domain by a suitable mapping function, which enables the imposition of far-field boundary conditions in discretised equations. The present chapter utilizes this approach in the finite difference solution of two dimentional quasi-steady conduction equation for continuous casting of an infinite slab...

3.2 GOVERNING EQUATION AND BOUNDARY CONDITIONS

3.2.1 Conduction equation for the slab

In conduction controlled continuous casting analysis ,axial conduction along the slab from solid region to liquid region is the dominant mechanism of heat removal ahead of the quench front.

The two dimensional transient conduction equation for the slab is

$$K\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}\right)=\rho C\frac{\partial T}{\partial t} \qquad 0 < X < L \quad 0 < Y < \delta , L \to \infty \qquad \text{---3.1}$$

Where L is the length of the slab and d is the thickness of the slab. The density, specific heat and thermal conductivity of the slab material are ρ, C, and K respectively. The origin of the coordinate frame is at left-bottom corner of the slab. To convert this transient equation into a quasi-steady state equation, the following transformation is used:

$$\bar{x} = X - ut \qquad\qquad \bar{y} = Y$$

Where u is the constant quench front velocity and x^{-}, y^{-} are normal and axial coordinates respectively .Experiments have shown that,. That is, an observer stationed at the origin of the moving (x^{-},y^{-}) coordinate system fails to notice any appreciable change in the temperature distribution around him as the front moves on. This is identified as the apparent steady state or quasi-steady state condition.

The two dimensional conduction equations convert in nondimensional form of conduction equation

$$\frac{\partial T}{\partial Y} = \frac{\partial T}{\partial \overline{y}} * \frac{\partial \overline{y}}{\partial Y} + \frac{\partial T}{\partial \tau} * \frac{\partial \tau}{\partial Y} = \frac{\partial T}{\partial \overline{y}} \qquad \text{------------3.2a}$$

Similarly

$$\frac{\partial^2 T}{\partial Y^2} = \frac{\partial^2 T}{\partial \overline{y^2}} \qquad \text{------------3.2b}$$

The x-coordinate can be transformed to $\overline{x}$-coordinate by substituting $\overline{x} = X$

$$\frac{\partial^2 T}{\partial X^2} = \frac{\partial^2 T}{\partial \overline{x^2}} \qquad \text{------------3.2c}$$

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \overline{x}} * \frac{\partial \overline{x}}{\partial t} = -u \frac{\partial T}{\partial \overline{x}} \qquad \text{------------3.2d}$$

Substituting these partial derivatives of equation into conduction equation

$$K\left(\frac{\partial^2 T}{\partial \overline{x^2}} + \frac{\partial^2 T}{\partial \overline{y^2}}\right) = -u\rho c \frac{\partial T}{\partial \overline{x}} \qquad \text{--------------3.3}$$

$$K\left(\frac{\partial^2 T}{\partial \overline{x^2}} + \frac{\partial^2 T}{\partial \overline{y^2}}\right) + u\rho c \frac{\partial T}{\partial \overline{x}} = 0 \qquad \text{--------------3.4}$$

3.2.2 Non – Dimensionalization

In order to obtain a solution for the general applicability, the governing equation and its boundary conditions are non-dimensional with following variables:

Let $U_1 = \frac{u\delta}{\alpha_1}$ = Peclet No (Solid) , $U_2 = \frac{u\delta}{\alpha_2}$ = Peclet No (Liquid)

$$\theta_1 = \frac{T_1 - T_c}{T_l - T_c},\ \theta_2 = \frac{T_2 - T_c}{T_l - T_c},\ x = \frac{\bar{x}}{\delta},\ y = \frac{\bar{y}}{\delta},\ \text{Bi}= \frac{h\delta}{k},\ \text{Q}=\frac{UL\rho\delta}{k(T_l - T_c)} \qquad \text{-----------3.5}$$

where x and y are normal and axial coordinates, B_1 and B_2 are solid side and liquid side Biot numbers , U_1 and U_2 is Peclet number and Q is dimensionless heat source parameter, θ is the non dimentional temperature .

Differentiate the temperature

$$\frac{\partial\theta}{\partial x} = \frac{1}{(T_l - T_c)}\frac{\partial T}{\partial x} \qquad \frac{\partial\theta}{\partial \bar{x}} = \frac{1}{(T_l - T_c)}\frac{\partial T}{\partial \bar{x}} \qquad \text{---------------------3.6a}$$

$$\frac{1}{\delta}\frac{\partial\theta}{\partial x} = \frac{1}{(T_l - T_c)}\frac{\partial T}{\partial x} \qquad \text{---------------------3.6b}$$

$$\frac{\partial\theta}{\partial x} = \frac{\delta}{(T_l - T_c)}\frac{\partial T}{\partial \bar{x}} \qquad \text{---------------------3.6c}$$

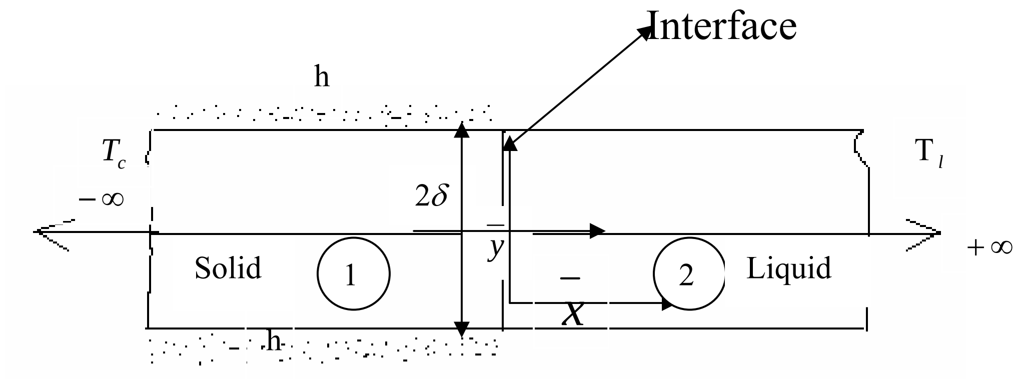
$$\frac{\partial^2\theta}{\partial x^2} = \frac{\delta^2}{(T_l - T_c)}\frac{\partial^2 T}{\partial \bar{x}^2} \qquad \text{---------------------3.6d}$$

Substituting these partial derivatives of equation into conduction equation

We get conduction equation in Non dimensional form

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + Pe\ \frac{\partial\theta}{\partial x} = 0 \qquad Pe = \frac{u\delta}{\alpha} \qquad \text{-----------------------3.7}$$

3.2.3 Governing Equations in Moving Coordinate



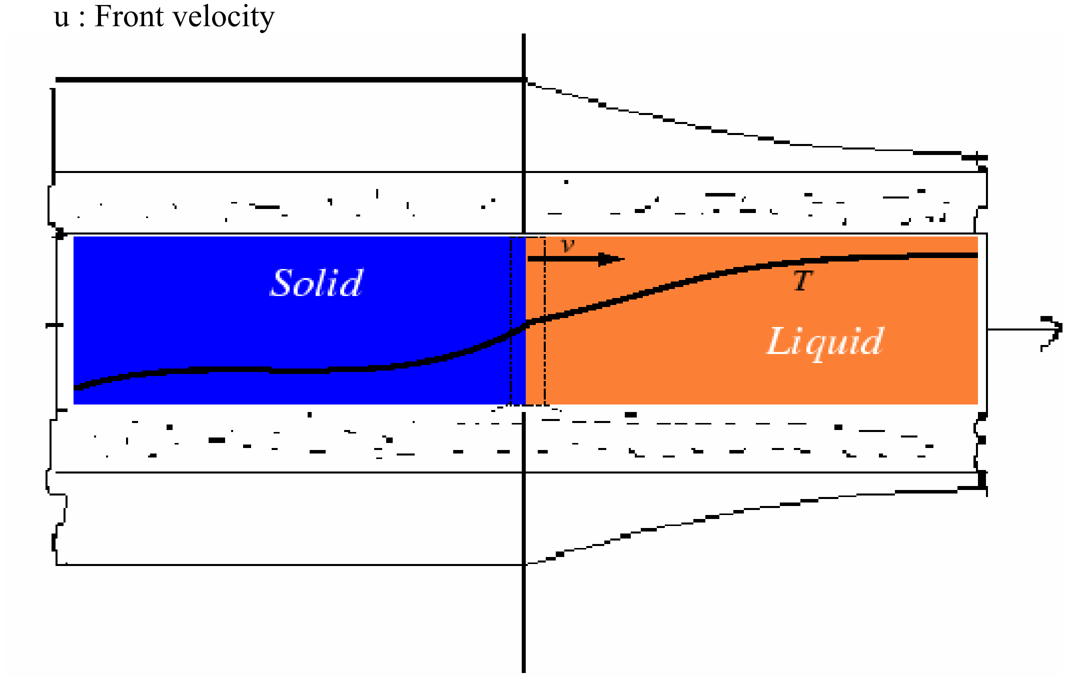
T_c : Coolant Temperature

T_l : Liquid (hot metal) Temperature

T_m : Melting (interface)Temperature

h : Heat transfer coefficient

u : Front velocity



$$\frac{\partial^2 T_1}{\partial \bar{x}^2} + \frac{\partial^2 T_1}{\partial \bar{y}^2} + \frac{u}{\alpha_1}\frac{\partial \bar{T}_1}{\partial x} = 0$$ ---------------------3.8a

$$\frac{\partial^2 T_2}{\partial \bar{x}^2} + \frac{\partial^2 T_2}{\partial \bar{y}^2} + \frac{u}{\alpha_2}\frac{\partial \bar{T}_2}{\partial x} = 0$$ ---------------------3.8b

3.2.4 BOUNDARY CONDITION

At $\bar{x} \to -\infty, T_1 = T_c$ ---------------------3.9a

At $\bar{x} \to +\infty, T_2 = T_l$ ---------------------3.9b

At $\bar{x} \to 0, T_1 = T_2 = T_m$ ---------------------3.9c

Let $\bar{y} = 0$, $\kappa_1 \frac{\partial T_1}{\partial y} = h(T_1 - T_c)$ ---------------------3.9d

Let $\bar{y} = 0$, $\kappa_1 \frac{\partial T_1}{\partial \bar{y}} = 0$ ---------------------3.9e

Let $\bar{y} = \delta$, $\kappa_1 \frac{\partial T_1}{\partial \bar{y}} = \kappa_2 \frac{\partial T_2}{\partial \bar{y}} = 0$ ---------------------3.9f

3.2.5 Stefan's condition at interface

$$\kappa_1 \frac{\partial T_1}{\partial \bar{x}} - \kappa_2 \frac{\partial T}{\partial \bar{x}} = L\mu(\rho_1 - \rho_2)$$ --------------------3.10a

If density of fluid neglected

$$\kappa_1 \frac{\partial T_1}{\partial \bar{x}} - \kappa_2 \frac{\partial T}{\partial \bar{x}} = \rho_1 L\mu$$ --------------------3.10b

3.3 Governing Equations

$$\frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2} + U_1 \frac{\partial \theta_1}{\partial x} = 0 \quad , \qquad -\infty < x < 0$$ ----------------3.11a

$$\frac{\partial^2 \theta_2}{\partial x^2} + \frac{\partial^2 \theta_2}{\partial y^2} + U_2 \frac{\partial \theta_2}{\partial x} = 0 \; , \qquad -0 < x < \infty$$ ----------------3.11b

<u>Where</u>

$$\theta_m = \frac{T_1 - T_C}{T_l - T_C} \quad , \quad \theta_m = \frac{T_2 - T_C}{T_l - T_C}$$

$$U_1 = \frac{u\delta}{\alpha_1} = \text{Peclet No (Solid)},$$

$$U_2 = \frac{u\delta}{\alpha_2} = \text{Peclet No (Liquid)}$$

3.3.1Boundary Conditions

Let $x \to -\infty, \theta_1 = 0$ --------------------3.12a

$x \to +\infty, \theta_2 = 1$ --------------------3.12b

Let y = 0 , $\frac{\partial \theta_1}{\partial y} = B_i \theta_l , \frac{\partial \theta_2}{\partial y} = 0$ --------------------3.12c

Let y = 1, $\frac{\partial \theta_1}{\partial y} = \frac{\partial \theta_2}{\partial y} = 0$ --------------------3.12d

Let x = 0, $\theta_1 = \theta_2$ and $\frac{\partial \theta_1}{\partial x} = \Gamma \frac{\partial \theta_2}{\partial x} + Q$ --------------------3.12e

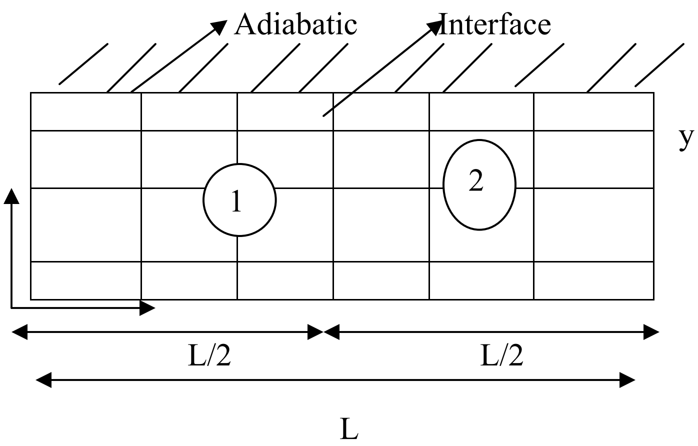
Where $\Gamma = \frac{k_2}{k_1}$, Q = $\frac{\rho_1 L \mu \delta}{k_4 (T_l - T_c)}$

Where: Q is dimensionless heat source

L is latent heat of fusion.

Γ is conductivity Ratio

3.3.2 Computational Geometry and Finite Difference Equation



3.3.3 Governing Equation

$$\frac{\partial^2\theta_1}{\partial x^2}+\frac{\partial^2\theta_1}{\partial y^2}+U_1\frac{\partial\theta_1}{\partial x}=0\text{ , } 0< x < L/2 \qquad \text{--------------------3.13a}$$

$$\frac{\partial^2\theta_2}{\partial x^2}+\frac{\partial^2\theta_2}{\partial y^2}+U_2\frac{\partial\theta_2}{\partial x}=0\text{ , } L/2 < x <L \qquad \text{--------------------3.13b}$$

3.3.4 Boundary Conditions

Let $\mathrm{x}=0,\theta_1=0$

$\mathrm{x}=L,\theta_2=1$

Let y = 0, $\dfrac{\partial\theta_1}{\partial y}=B_i\theta_l\text{ , }\dfrac{\partial\theta_2}{\partial y}=0$

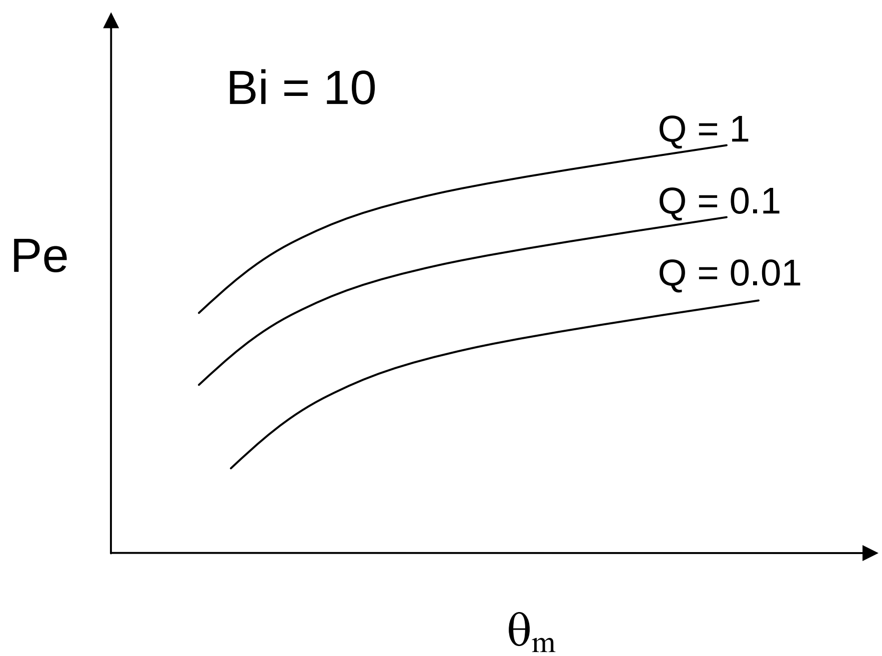
Let y = 1 , $\dfrac{\partial\theta_1}{\partial y}=\dfrac{\partial\theta_2}{\partial y}=0$

Let x = L/2, $\theta_1=\theta_2$ and $\dfrac{\partial\theta_1}{\partial x}=\Gamma\dfrac{\partial\theta_2}{\partial x}+Q$

For given values of U_1, U_2, Γ, Q and B_i, Find θ_m

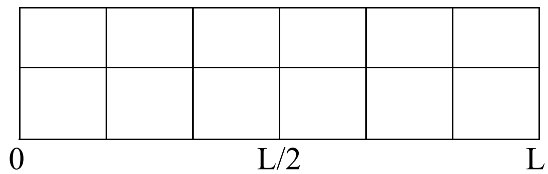
$T_m = f(u)$

Assuming $U = \dfrac{2u_1u_2}{u_1 + u_2}$, Plot Pe Vs θ_m



Also Plot the Temperature Distribution

3.3.5 Grid clustering at an interior Plane

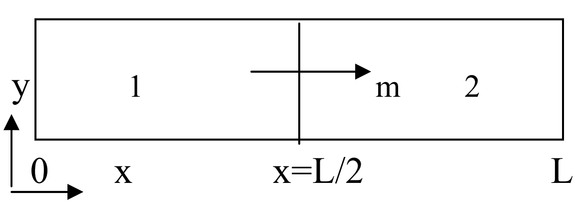


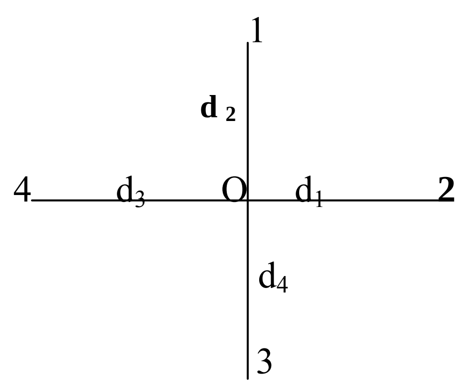
$$x = L/2\{1 + \frac{\sinh[\tau(\xi - 0.5)]}{\sinh(0.5\tau)}\} \text{ , } 0 < \tau < \infty$$

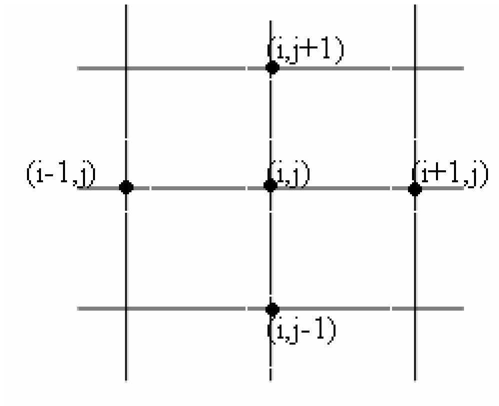
In this transformation τ is the stretching parameter which varies from zero (no stretching) to large values which produce refinement mean x=L/2.

3.4 Finite –Difference Equations

_In a finite difference method ,the continuous domain is discretized so that the dependent variables are considered to exit only at discrete points. Derivatives are approximated by difference ,resulting in an algebraic representation of partial differential equation (PDE). The five point finite difference representation of elliptic PDE can be written in the general form







$$A^0_{i,j}\theta_{i,j} = A^1_{i,j}\theta_{i,j+1} + A^2_{i,j}\theta_{i+1,j} + A^3_{i,j}\theta_{i,j-1} + A^4_{i,j}\theta_{i-1,j} + S_{i,j}$$

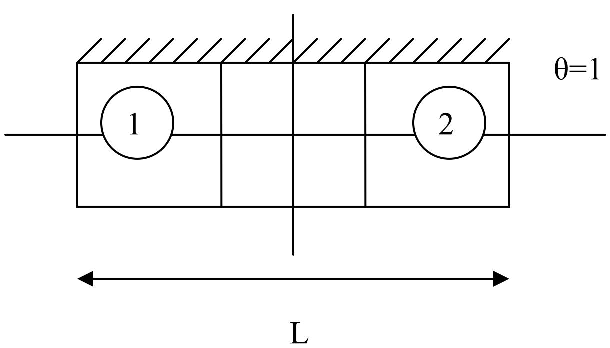
Where sub indices i and j represent the normal and axial direction respectively .The values of coefficients and source term of the difference equation are determined by control volume for formulation.The differential equation is integrated over control volume.

By this method ,the calculation domain is divided into a finite number of Non-overlapping control volumes such that there is only one control volume surrounding each grid point. The differential equation is integrated over each control volume.

3.4.1 Finite –Difference Approach

Presently, the Finite difference method (FDM) is widely used for the solution of partial differential equations of heat, mass and momentum transfer. FDM, Finite volume method (FVM), and Finite element method (FEM), each method has its advantages depending on the nature of physical problem to be solved; but there is no best method for all problems. The accuracy of the FDM can readily be examined by the order of the truncation error in the Taylor series expansion. The dimension of the problem is another factor that deserves some consideration. For example, an efficient method for 1-D problems may not be so efficient for 2-D, or 3-D problems. FDM are simple to formulate and can readily be extended to two or three dimensional problems and require less computational work than the FEM. Furthermore, FDM is very easy to learn and apply for the solution of partial differential equations encountered in the modeling of engineering problems for simple geometries (i.e., not very irregular). For problems involving irregular geometries FDM may not be so suitable. A major drawback of FDM appears to be in its inability to handle effectively the solution of problems over arbitrarily shaped complex geometries because of interpolation difficulties between the boundaries and the interior points in order to develop finite difference expressions for nodes next to the boundaries. More recently, with the advent of numerical grid generation approach, the FDM has become comparable to FEM in dealing with irregular geometries, while still maintaining the simplicity of the standard finite difference method.

The basic philosophy of finite difference methods is to replace the derivatives of the governing equations with algebraic difference quotients. This will result in a system of algebraic equations which can be solved for the dependent variables at the discrete grid points in the flow field.



$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + U \ \frac{\partial \theta_1}{\partial x} = 0 \qquad \text{-----------3.14a}$$

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{2}{d_1 + d_3}\left[\frac{\theta_{i+1,j} - \theta_{i,j}}{d_1} - \frac{\theta_{i,j} - \theta_{i-1,j}}{d_3}\right] \qquad \text{--------------------3.14b}$$

$$\frac{\partial^2\theta}{\partial y^2}=\frac{2}{d_2+d_4}\left[\frac{\theta_{i,j+1}-\theta_{i,j}}{d_2}-\frac{\theta_{i,j}-\theta_{i,j-1}}{d_4}\right] \qquad \text{-------------------- 3.14c}$$

$$\frac{\partial\theta}{\partial x}=\frac{1}{d_1+d_3}\left[\frac{d_3}{d_1}(\theta_{i+1,j}-\theta_{i,j})-\frac{d_1}{d_3}(\theta_{i-1,j}-\theta_{i.j})\right] \qquad \text{---------------------3.14d}$$

3.4.2 Solid Region (Region 1)

<u>Internal Node</u>

$$\frac{\partial^2\theta_1}{\partial x^2}+\frac{\partial^2\theta_1}{\partial y^2}+U_1\frac{\partial\theta_1}{\partial x}=0 \qquad \text{---------------------3.15a}$$

$$\frac{2}{d_1+d_3}\left[\frac{\theta_{i+1,j}-\theta_{i,j}}{d_1}-\frac{\theta_{i,j}-\theta_{i-1,j}}{d_3}\right]+\frac{2}{d_1+d_4}\left[\frac{\theta_{j+1}-\theta_{i,j}}{d_2}-\frac{\theta_{i,j}-\theta_{i,j-1}}{d_4}\right]+$$
$$U_1\left[\frac{\theta_{i+1,j}-\theta_{i,j}}{d_1}\right]=0 \qquad \text{------------------------3.15b}$$

Rearranging the terms, after the coefficients are found as

$$A^1_{i,j}=\frac{2}{d_2(d_2+d_4)}\quad,\quad A^2_{i,j}=\frac{1}{(d_1+d_3)}\left[\frac{1}{d_1}+\frac{U_1}{2}\right]$$
$$A^3_{i,j}=\frac{2}{d_4(d_2+d_4)}\quad,\quad A^4_{i,j}=\frac{2}{d_3(d_1+d_3)}$$
$$A^0_{i,j}=\sum_{K=1}^{4}A^K{}_{i,j}$$

<u>Boundary Node(y=1)</u>

$$\frac{\partial^2\theta_1}{\partial x^2}+\frac{\partial^2\theta_1}{\partial y^2}+U_1\frac{\partial\theta_1}{\partial x}=0 \qquad \text{-----------3.16a}$$

$$\frac{2}{d_1+d_3}\left[\frac{\theta_{i+1,j}-\theta_{i,j}}{d_1}-\frac{\theta_{i,j}-\theta_{i-1,j}}{d_3}\right]+\frac{2}{2d_4}\left[\frac{2\theta_{i,j-1}-2\theta_{i,j}}{d_4}\right]+\quad U_1\left[\frac{\theta_{i+1,j}-\theta_{i,j}}{d_1}\right]=0$$

and rearranging the terms ,the coefficients are found as

$$A^1_{i,j}=0\ ,\quad A^2{}_{i,j}=\frac{2}{d_1(d_1+d_3)}+\frac{U_1}{d_1}\ ,\quad A^3_{i,j}=\frac{1}{d_4{}^2}$$

$$A^4{}_{i,j}=\frac{1}{d_3(d_1+d_3)}\quad ,\quad A^0_{i,j}=\sum_{K=1}^{4}A^K{}_{i,j}$$

<u>Boundary Node (y = 0)</u>

$$\frac{\partial\theta}{\partial y}=B_i\theta \qquad \text{------------------3.17a}$$

$$\frac{\theta_{i,j+1}-\theta_{i,j-1}}{2d_2}=B_i\theta_{i,j} \qquad \text{--------------------3.17b}$$

$$\frac{\partial^2\theta_1}{\partial x^2}+\frac{\partial^2\theta_1}{\partial y^2}+U_1\frac{\partial\theta_1}{\partial x}=0 \qquad \text{--------------------3.17c}$$

$$\frac{2}{d_1+d_3}\left[\frac{\theta_{i+1,j}-\theta_{i,j}}{d_1}-\frac{\theta_{i,j}-\theta_{i-1,j}}{d_3}\right]+\frac{2}{2d_2}\left[\frac{\theta_{i,j+1}-\theta_{i,j}}{d_2}-\frac{(\theta_{i,j}-\theta_{i,j+1}+2B_id_2\theta_{i,j})}{d_2}\right]$$

$$+\ U_1\left[\frac{\theta_{i+1,j}-\theta_{i,j}}{d_1}\right]=0 \qquad \text{--------------------3.17d}$$

and rearranging the terms ,the coefficients are found as

$$A^1_{i,j}=\frac{1}{d_2{}^2}\ ,\quad A^2_{i,j}=\frac{1}{(d_1+d_3)}\left[\frac{1}{d_1}+\frac{U_1}{2}\right]$$

$$A^3_{i,j}=0\quad ,\quad A^4{}_{i,j}=\frac{1}{d_3(d_1+d_3)}$$

$$A^0_{i,j}=\sum_{K=1}^{4}A^K{}_{i,j}+\frac{2B_i}{d_2}$$

Similarly solve for liquid region and Interface and We get

3.4.3 Liquid Region (Region-2)

Internal Node

$$A_{i,j}^{1} = \frac{1}{d_2(d_2+d_4)}, \; A_{i,j}^{2} = \frac{1}{(d_1+d_3)}\left[\frac{1}{d_1}+\frac{U_2}{2}\right]$$

$$A_{i,j}^{3} = \frac{1}{d_4(d_2+d_4)}, A_{i,j}^{4} = \frac{1}{d_3(d_1+d_3)}$$

$$A_{i,j}^{0} = \sum_{K=1}^{4} A^{K}{}_{i,j}$$

Boundary Node (y = 1)

$$\mathrm{A}_{i,j}^{1} = 0, \; A_{i,j}^{2} = \frac{1}{(d_1+d_3)}\left[\frac{1}{d_1}+\frac{U_2}{2}\right]$$

$$A_{i,j}^{3} = \frac{1}{d_4^{2}}, \; A_{i,j}^{4} = \frac{1}{d_3(d_1+d_3)}$$

$$A_{i,j}^{0} = \sum_{K=1}^{4} A^{K}{}_{i,j}$$

Boundary Node (y = 0)

$$\mathrm{A}_{i,j}^{1} = \frac{1}{d_2^{2}}, \; A_{i,j}^{2} = \frac{1}{(d_1+d_3)}\left[\frac{1}{d_1}+\frac{U_2}{2}\right]$$

$$A_{i,j}^{3} = 0 \;, \; A_{i,j}^{4} = \frac{1}{d_3(d_1+d_3)}$$

$$A_{i,j}^{0} = \sum_{K=1}^{4} A^{K}{}_{i,j}$$

3.4.4 Interface

Internal Node

$$A_{i,j}^{1} = \frac{1}{d_2(d_2+d_4)},\ A_{i,j}^{2} = \frac{1}{(d_1+d_3)}\left[\frac{1-\Gamma}{d_1}+U\right]$$

$$A_{i,j}^{3} = \frac{1}{d_4(d_2+d_4)},\ A_{i,j}^{4} = 0$$

$$A_{i,j}^{0} = \sum_{K=1}^{4} A^{K}{}_{i,j},\ S_{i,j} = \frac{Q}{d_1+d_2},\ U = \frac{2U_1U_2}{U_1+U_2},\alpha = \frac{2\alpha_1\alpha_2}{\alpha_1+\alpha_2}$$

Boundary Node (y =1)

$$A^{1}_{i,j} = 0,\ A_{i,j}^{2} = \frac{1}{(d_1+d_3)}\left[\frac{1-\Gamma}{d_1}+U\right]$$

$$A_{i,j}^{3} = \frac{1}{d_4^{\,2}},\ A_{i,j}^{4} = 0$$

$$A_{i,j}^{0} = \sum_{K=1}^{4} A^{K}{}_{i,j},\ S_{i,j} = \frac{Q}{d_1+d_2},U=\frac{2U_1U_2}{U_1+U_2},\ \alpha=\frac{2\alpha_1\alpha_2}{\alpha_1+\alpha_2}$$

Boundary Node (y = 0)

$$A^{1}_{i,j} = \frac{1}{d_2^{\,2}},\ A_{i,j}^{2} = \frac{1}{(d_1+d_3)}\left[\frac{1-\Gamma}{d_1}+U\right]$$

$$A_{i,j}^{3} = 0,\ A_{i,j}^{4} = 0$$

$$A_{i,j}^{0} = \sum_{K=1}^{4} A^{K}{}_{i,j} + \frac{2B_i}{d_2(d_1+d_3)},\ S_{i,j} = \frac{Q}{d_1+d_3},U=\frac{2U_1U_2}{U_1+U_2},\alpha=\frac{2\alpha_1\alpha_2}{\alpha_1+\alpha_2}$$

For a example

Material Properties for any metal

$\alpha_1 = 10^{-5}$ m^2/s , $\alpha_2 = 10^{-6}$ m^2/s , h=10^5 KJ/Kg, $\rho_1 = 8000$ kg/m^3,
k_1 = 50 w/m-^0C , k_2 =5 w/m-^0C

h=10^4 w/m^2-^0C,
δ=50 mm , T_c = 100 ^0C,
u=0.001m/s T_l = 1600 ^0C,

Biot Number, hδ/k_1=(10^4x 50x10^{-3})/50 =10

Peclet Number, $Pe_1 = U_1$ = uδ/α_1 = (0.001x 50x10^{-3})/10^{-5}= 5

Peclet Number, $Pe_2 = U_2$ = uδ/α_2 = (0.001x 50x10^{-3})/10^{-6}= 50

Conductivity Ratio= Γ= K_2/K_1=0.1

Q = Dimensionless heat source parameter = $\dfrac{\rho_1 L \mu \delta}{K_1(T_l - Tc)}$

= 0.5

Table 3.1 Material Properties used in the calculations

Sl. No.	Metal Name	Density(ρ) Kg/m^3		Thermal Diffusibility, α (10^{6} m^2/s)		Latent heat of fusion (L) KJ/Kg	Conductivity (k) w/m.k		Temperature (^{0}k)		Heat Capacity (J/Kg. k)
		ρ_1	ρ_2	α_1	α_2		K_1	K_2	T_c	T_l	
1	Iron	7870	7450	20	7	289	80	33	300	1900	520
2	Copper	9000	8500	112	85	209	400	335	300	1400	390
3	Aluminium	2707	2550	95	67	397	240	220	300	900	910

3.5 Closure

Different methods are available to solve the governing equations, mainly The Finite Difference Methods (FDM), Finite Element Methods (FEM) and Finite Volume Methods (FVM). Here we are using the Finite difference methods (FDM) .Although a number of studies on continuous casting of a slab exist in the literature. In this chapter, a two-dimensional numerical analysis by a finite difference method has been successfully carried out. The governing equation and its boundary conditions are derived under quasi-steady state conditions. The infinite domain of the slab has been mapped to a finite computational domain to implement the boundary conditions in the difference equations.. In the next chapter we will discuss about the results obtained from this algorithm.

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Chapter-4

Results and Discussions

- **Introduction**
- **Temperature Contours**
- **Temperature Profile on interface side**
- **Temperature profile on coolant side**

Chapter – 4

Results and Discussions

Introduction

As explained earlier the Conduction equation solved using Finite Difference Method. The iterative method of solution is carried out in FORTRAN 90. In the analysis of numerical computation of the temperature field has been carried out with Biot number ranging from 0.1 to 100 ,and dimension less heat source term Q from 10^{-3} to 10^{-1} .For all ranges of Biot numbers and therefore used for present calculations.

4.1Temperature Contours

The temperature contours are plotted in fig 4.1,4.2 and 4.3 for different Biot number. It can be observed those figs 4.1 to 4.3 the isotherms are are densely packed near the interface. This brings out the fact that the temperature gradiant in axial direction increases towards the Interface.

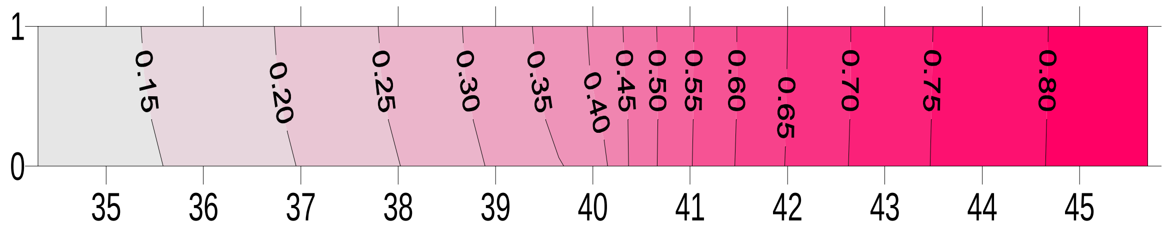
4.2 Temperature Profiles on the interface

The temperature profile s on the interface of the slab is shown in figs 4.4 to 4.6 .It can be observed from figs 4. 4 to 4.6 temperature gradient increases with increases in Biot number .This implies that conduction becomes significant as Biot number increases.

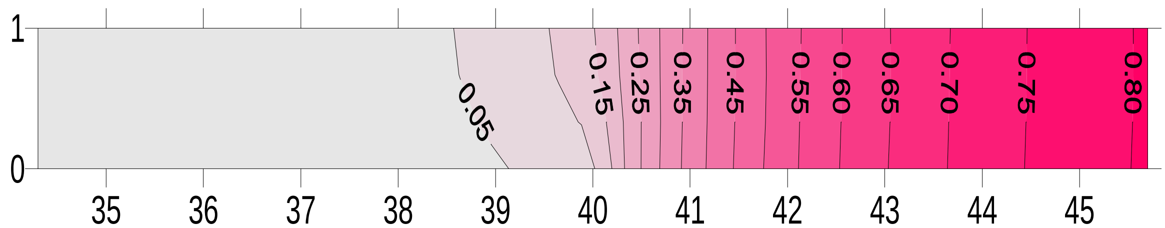
4.3 Temperature Profiles on the coolant side

The temperature profile s on the coolant side of the slab are shown in figs 4.7 to 4.9 .It can be observed from figs 4.7 to 4.9 temperature gradient increases with decreases in Biot number .This implies that axial conduction becomes significant as Biot number decreases.

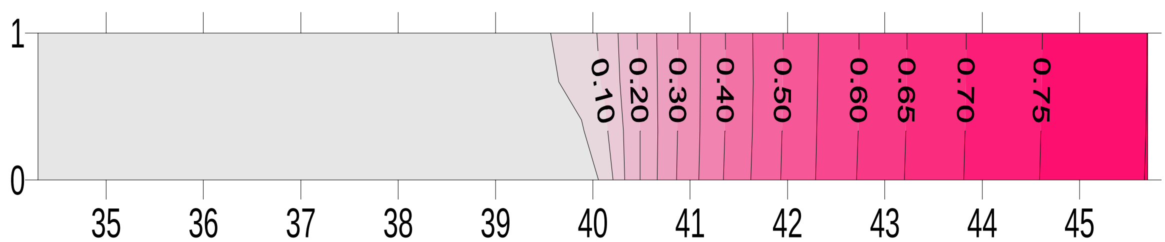
<u>FOR ALUMINIUM METAL</u>



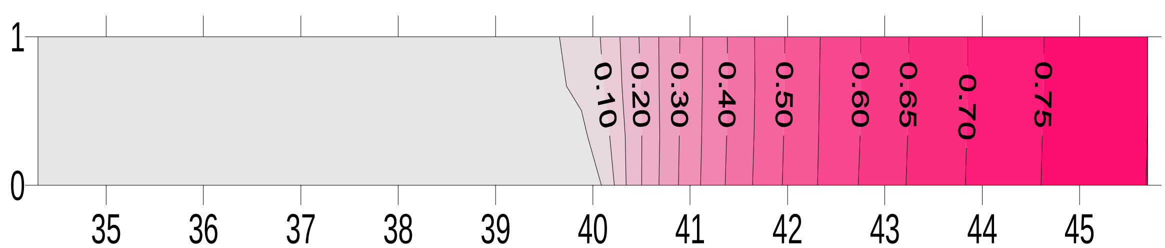
(a)



(b)



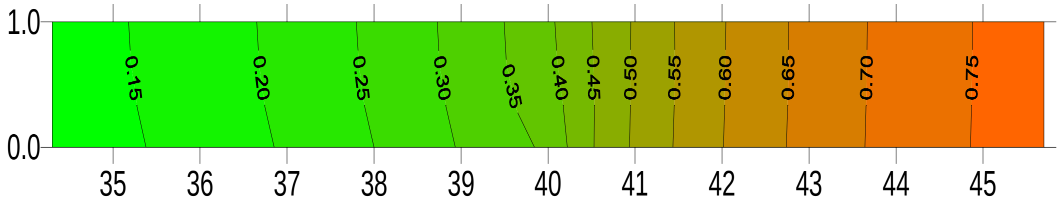
(c)



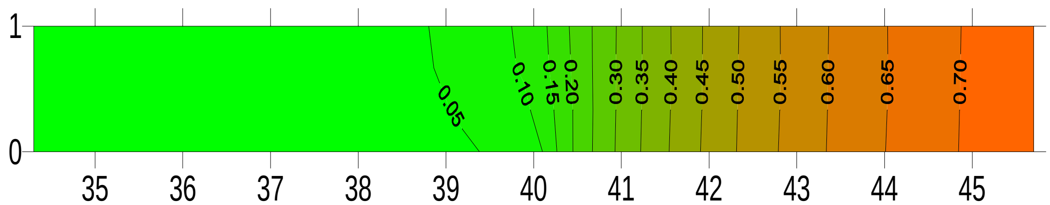
(d)

[Fig. 4.1] Temperature Contours in the slab (a) Bi=0.1 (b) Bi=1 (c)Bi=10 (d)Bi=100

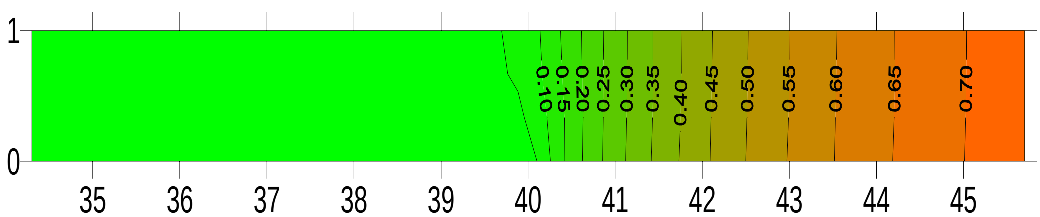
FOR COPPER METAL



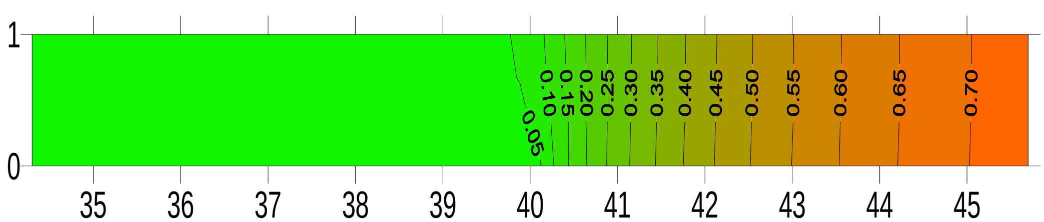
(a)



(b)



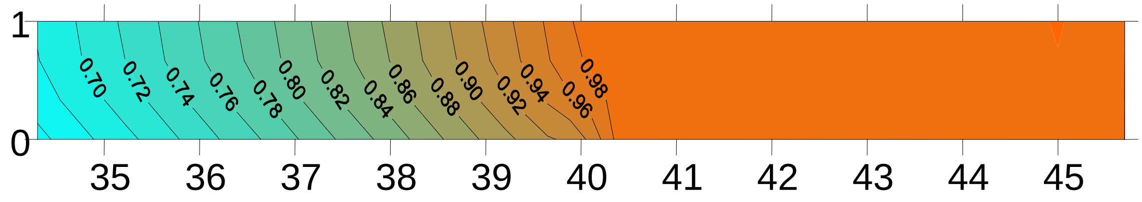
(c)



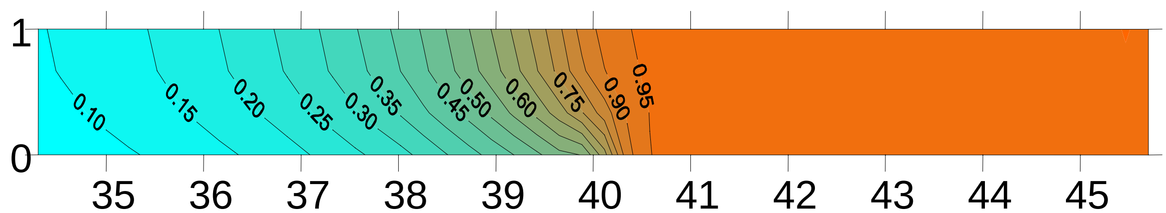
(d)

[Fig. 4.2] Temperature Contours in the slab (a)Bi=0.1 (b) Bi=1 (c)Bi=10 (d)Bi=100

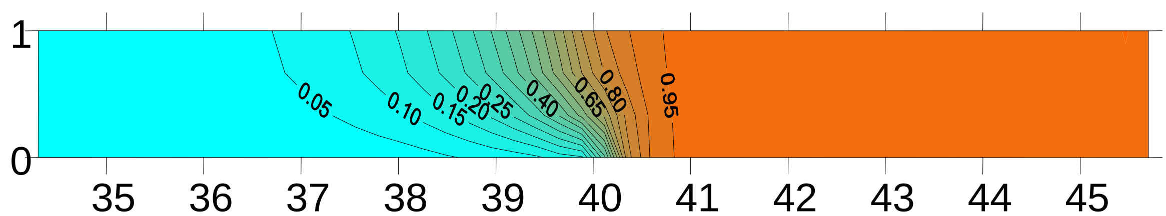
FOR IRON METAL



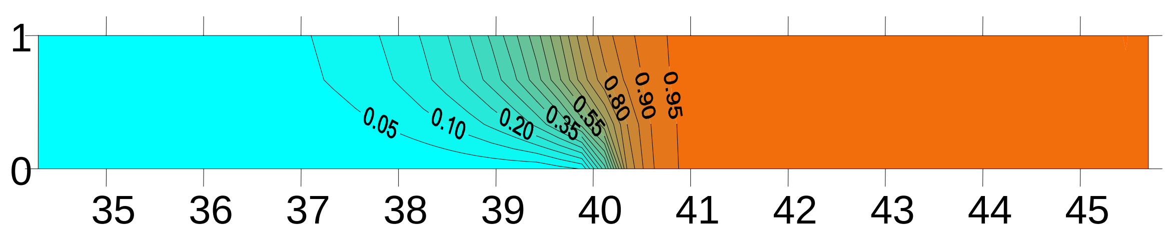
(a)



(b)



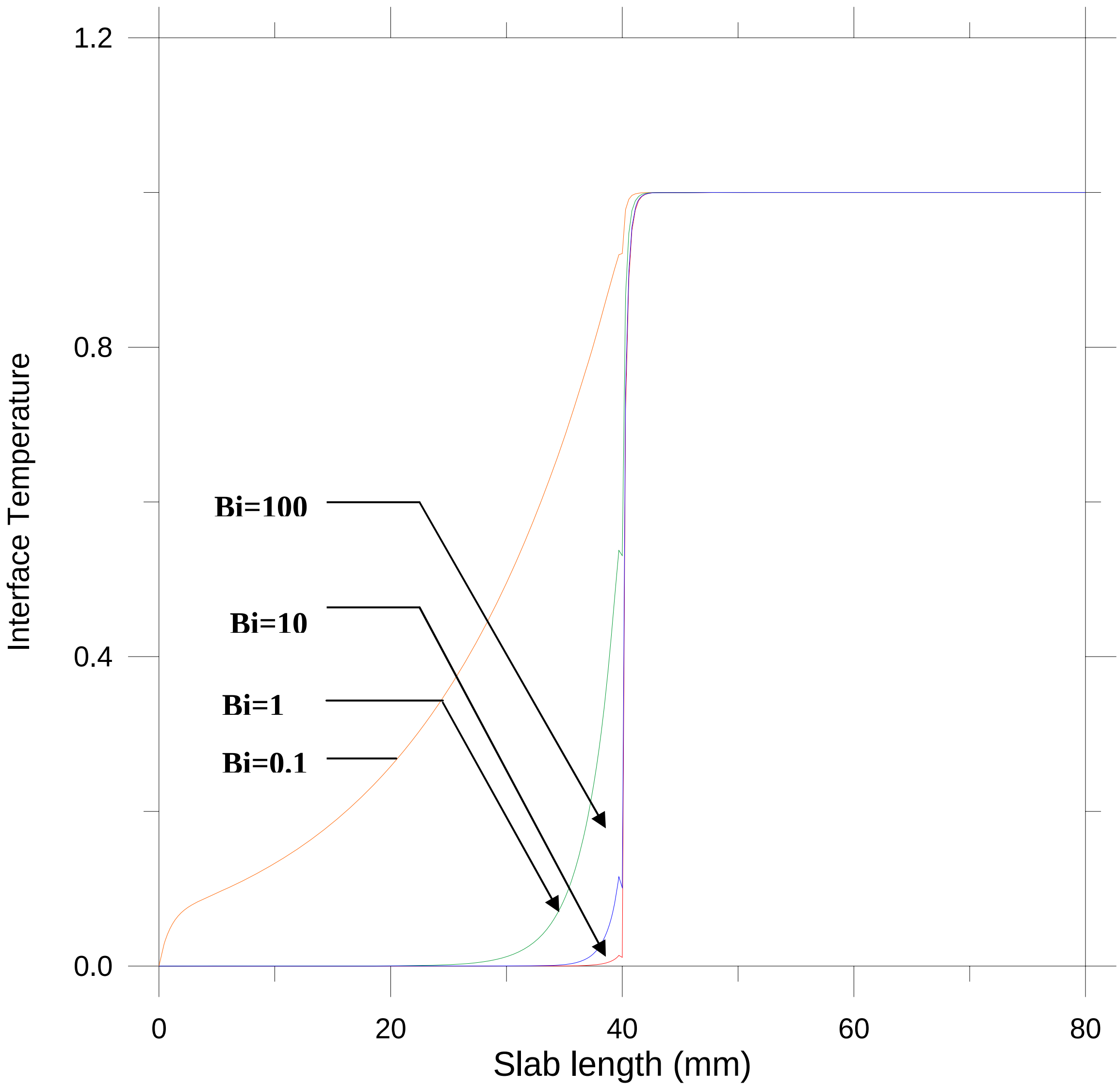
(c)



(d)

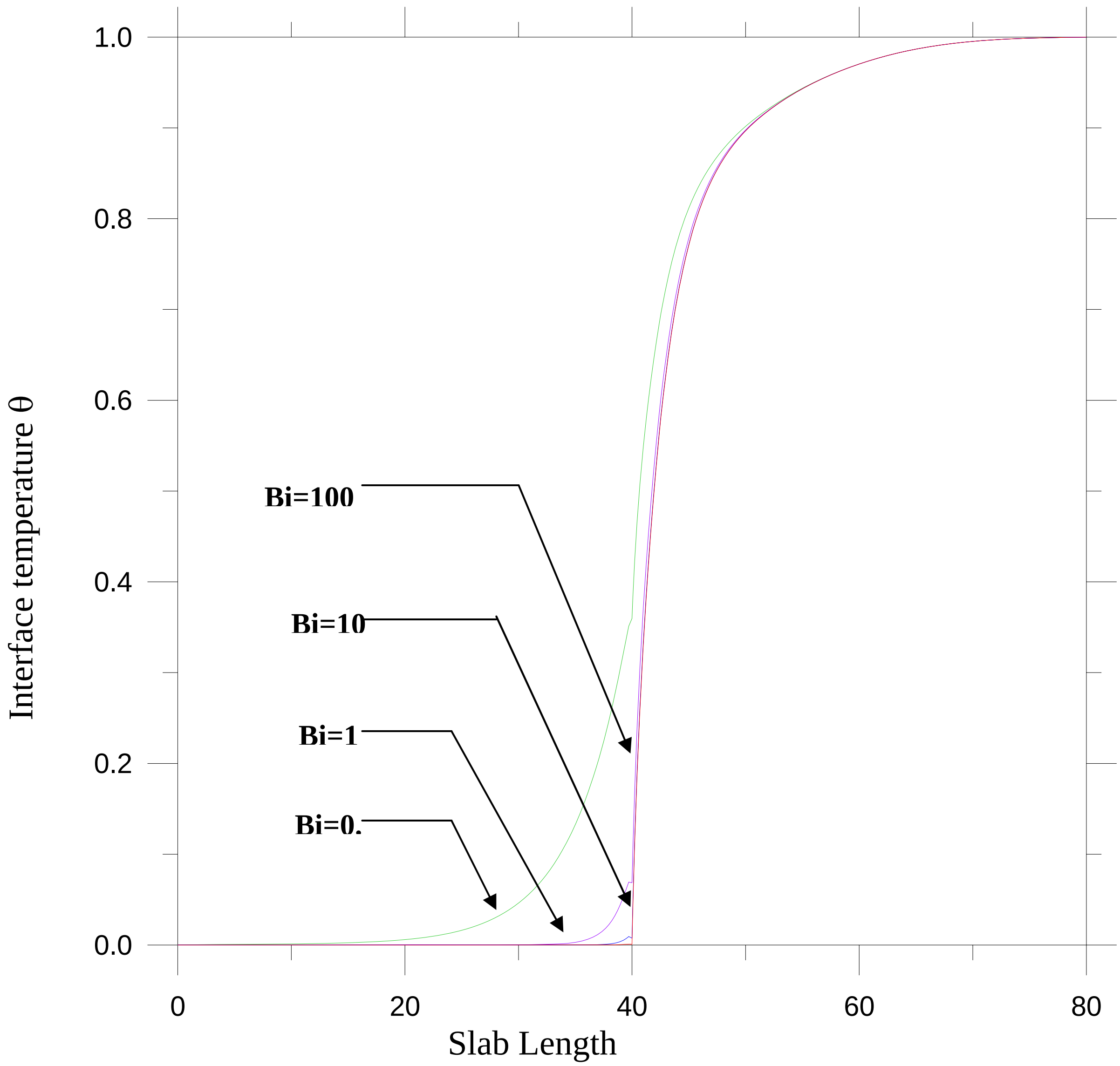
[Fig. 4.3] **Temperature Contours in the slab (a)Bi=0.1 (b) Bi=1 (c)Bi=10 (d)Bi=100**

FOR IRON METAL



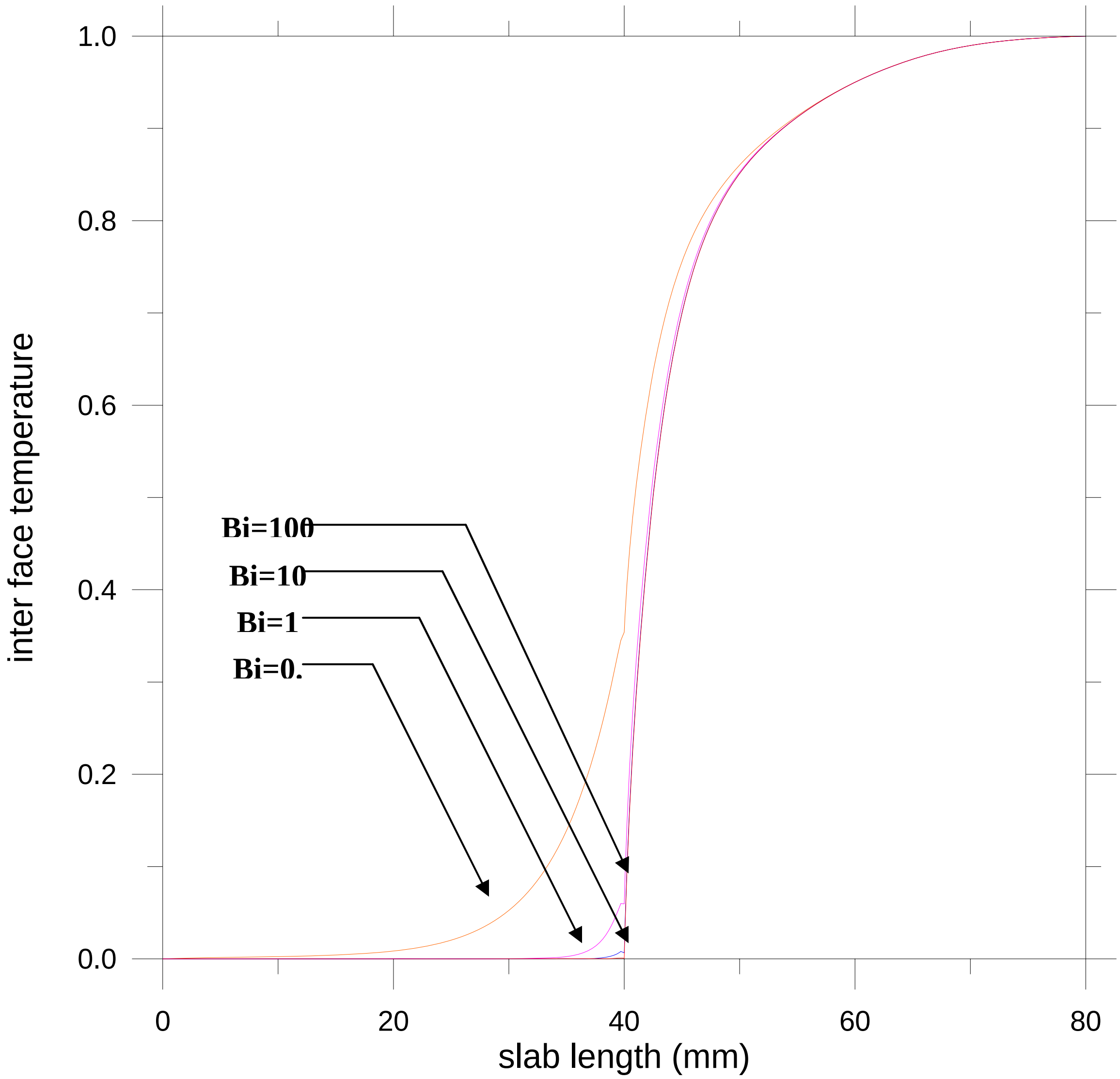
[Fig. 4.4] **Temperature profiles on the interface of the Iron slab at various Biot numbers**

FOR ALUMINIUM METAL



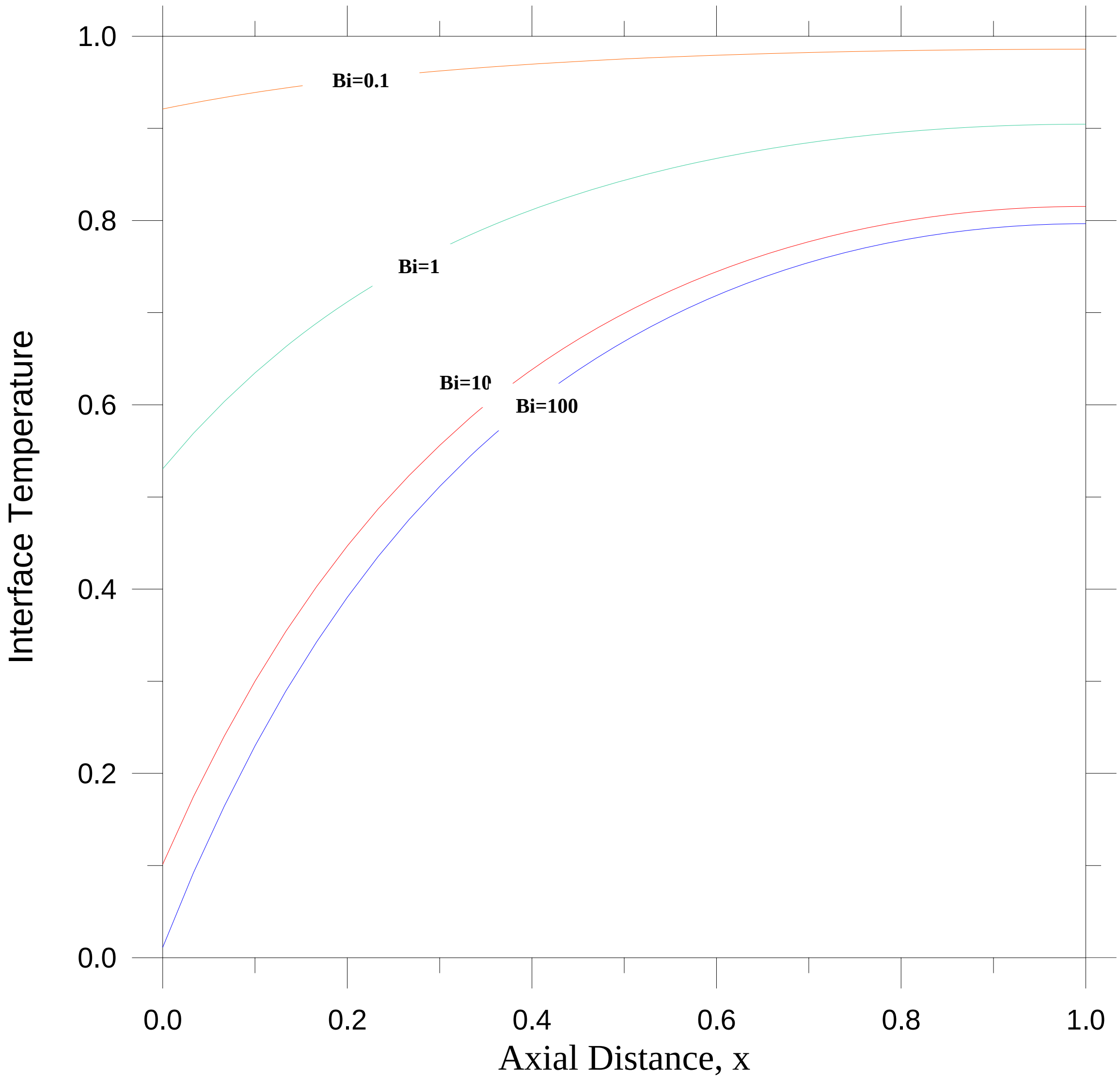
[Fig. 4.5] **Temperature profiles on the interface of the Aluminium slab at various Biot numbers**

FOR COPPEER METAL



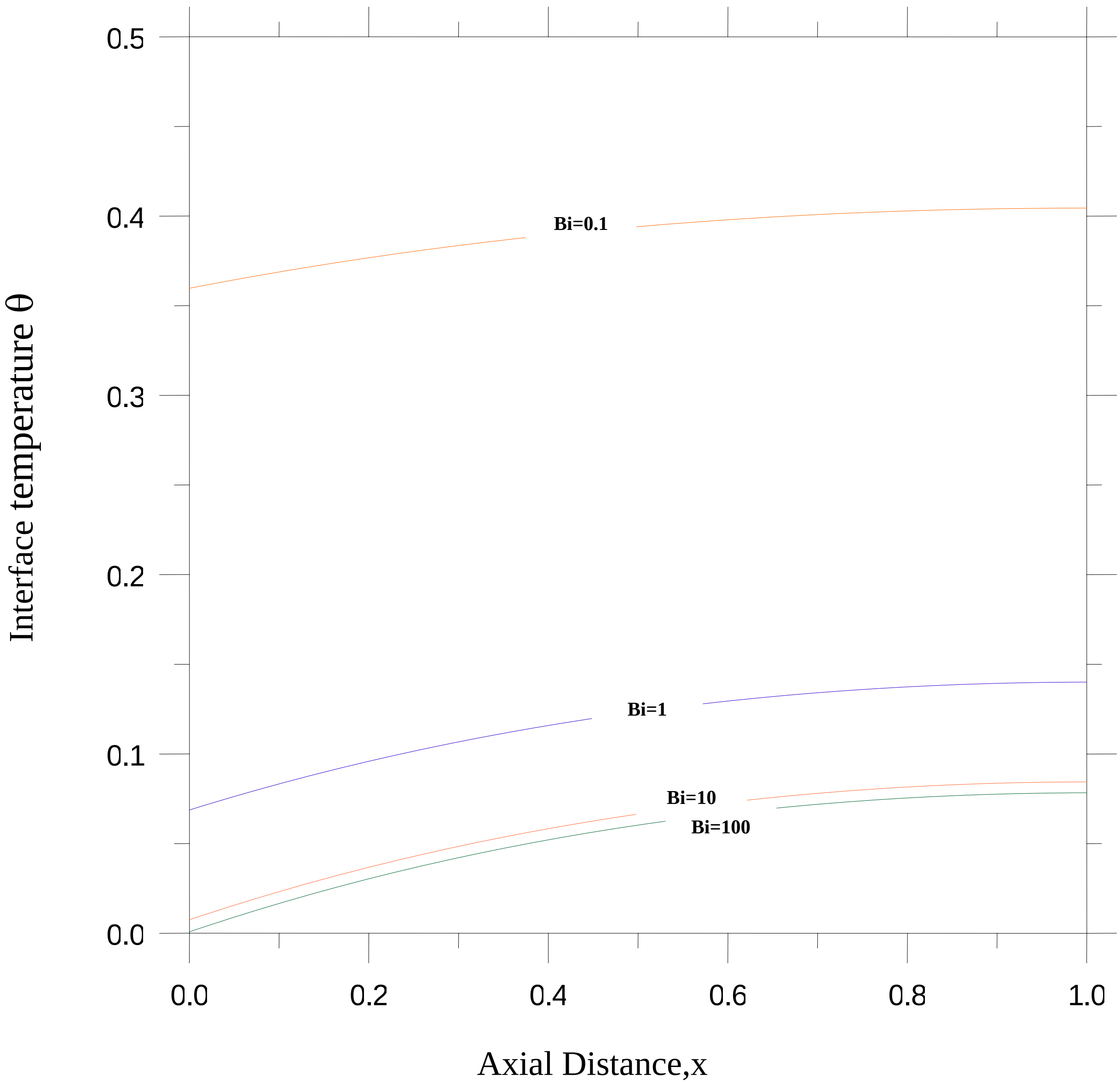
[Fig. 4.6] **Temperature profiles on the interface of the Copper slab at various Biot numbers**

FOR IRON METAL



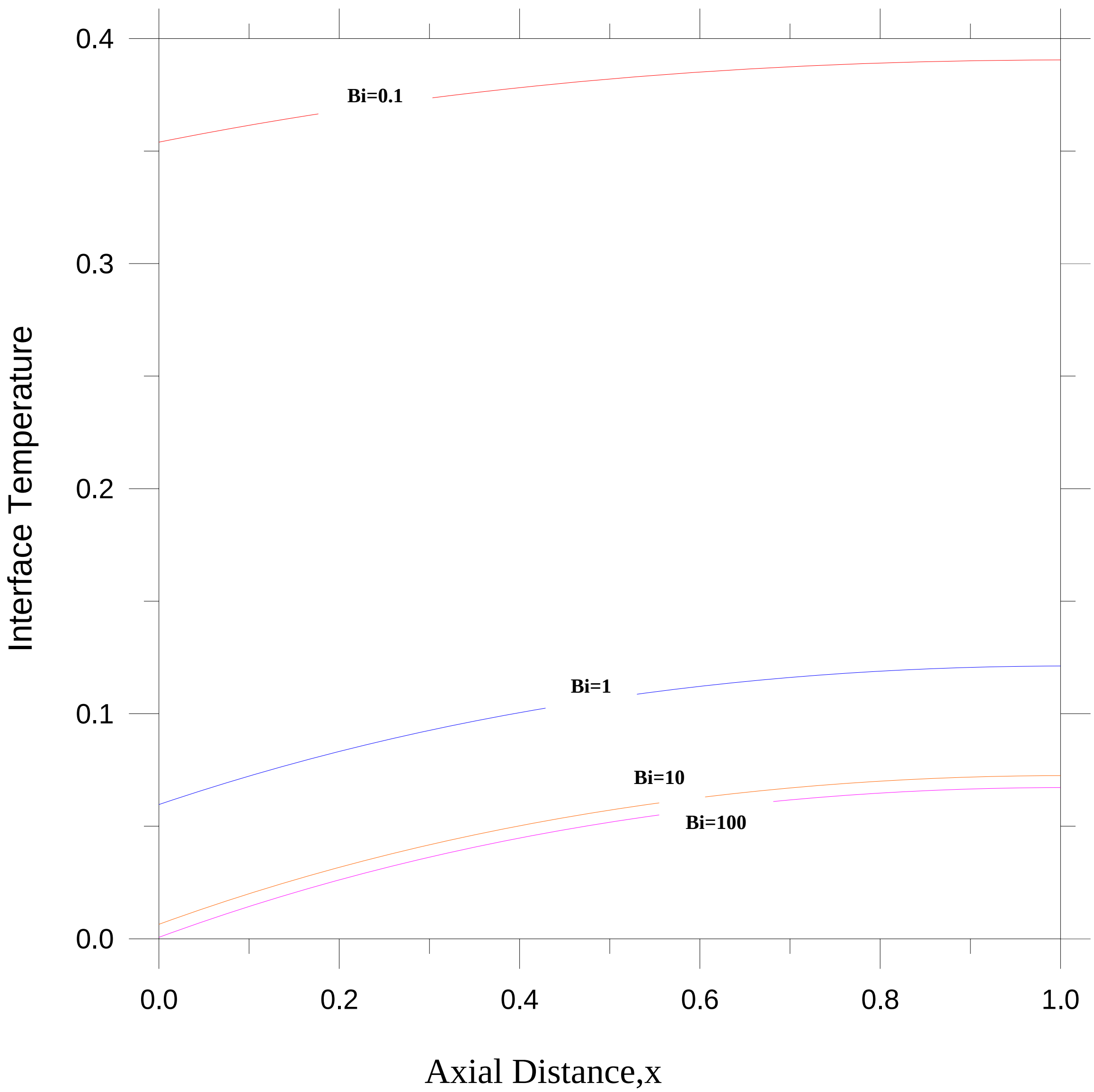
[Fig. 4.7] Temperature profiles on the coolant side of the Iron slab at various Biot numbers

FOR ALUMINIUM METAL



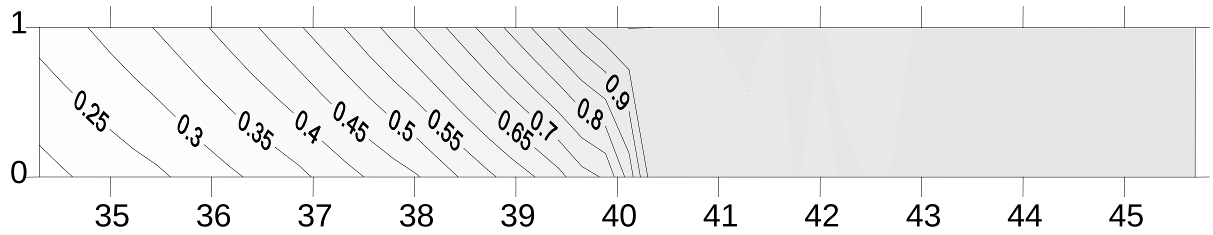
[Fig. 4.8] **Temperature profiles on the coolant side of the Aluminium slab at various Biot numbers**

FOR COPPER METAL

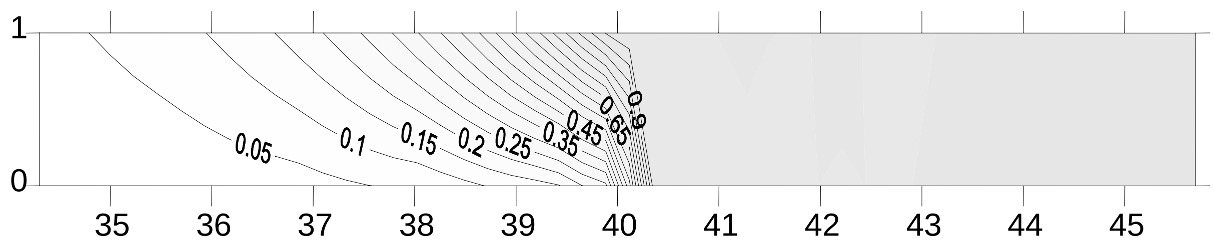


[Fig. 4.9] **Temperature profiles on the coolant side of the Copper slab at Various Biot numbers**

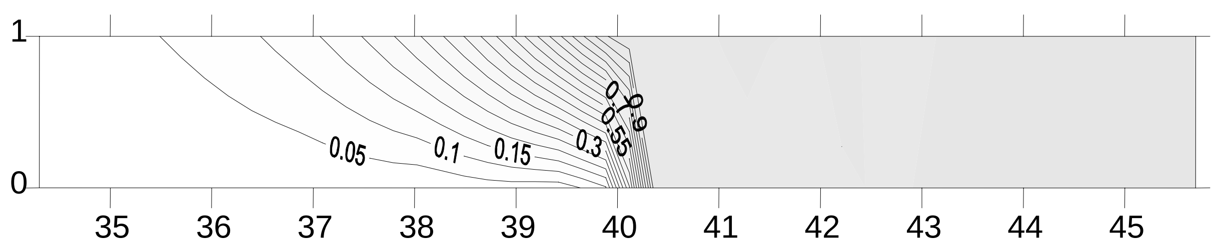
For a given metal in example



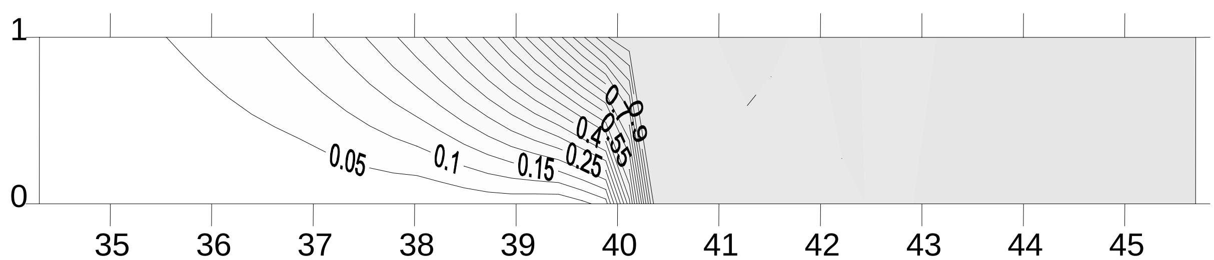
(a)



(b)



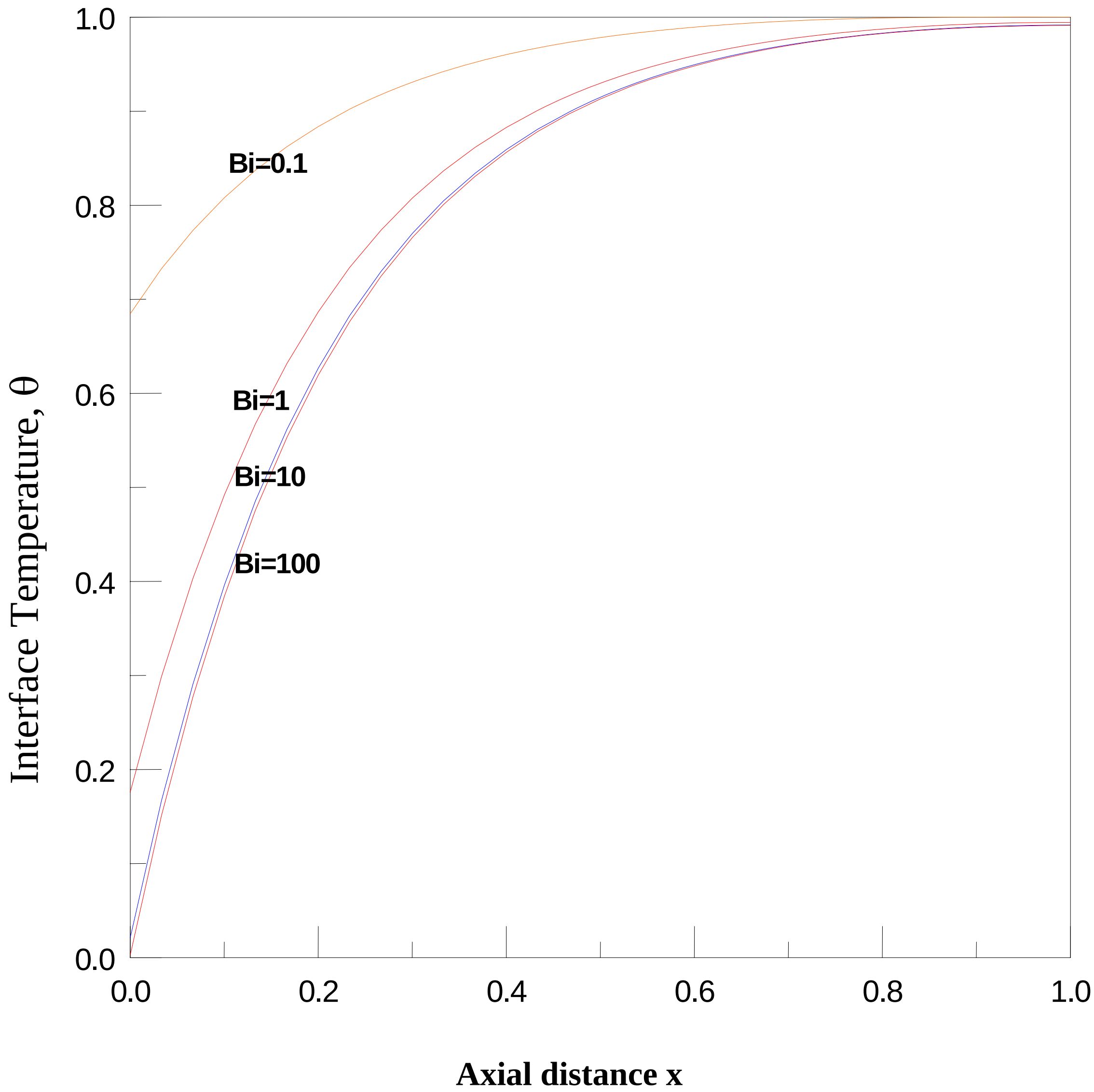
(c)



(d)

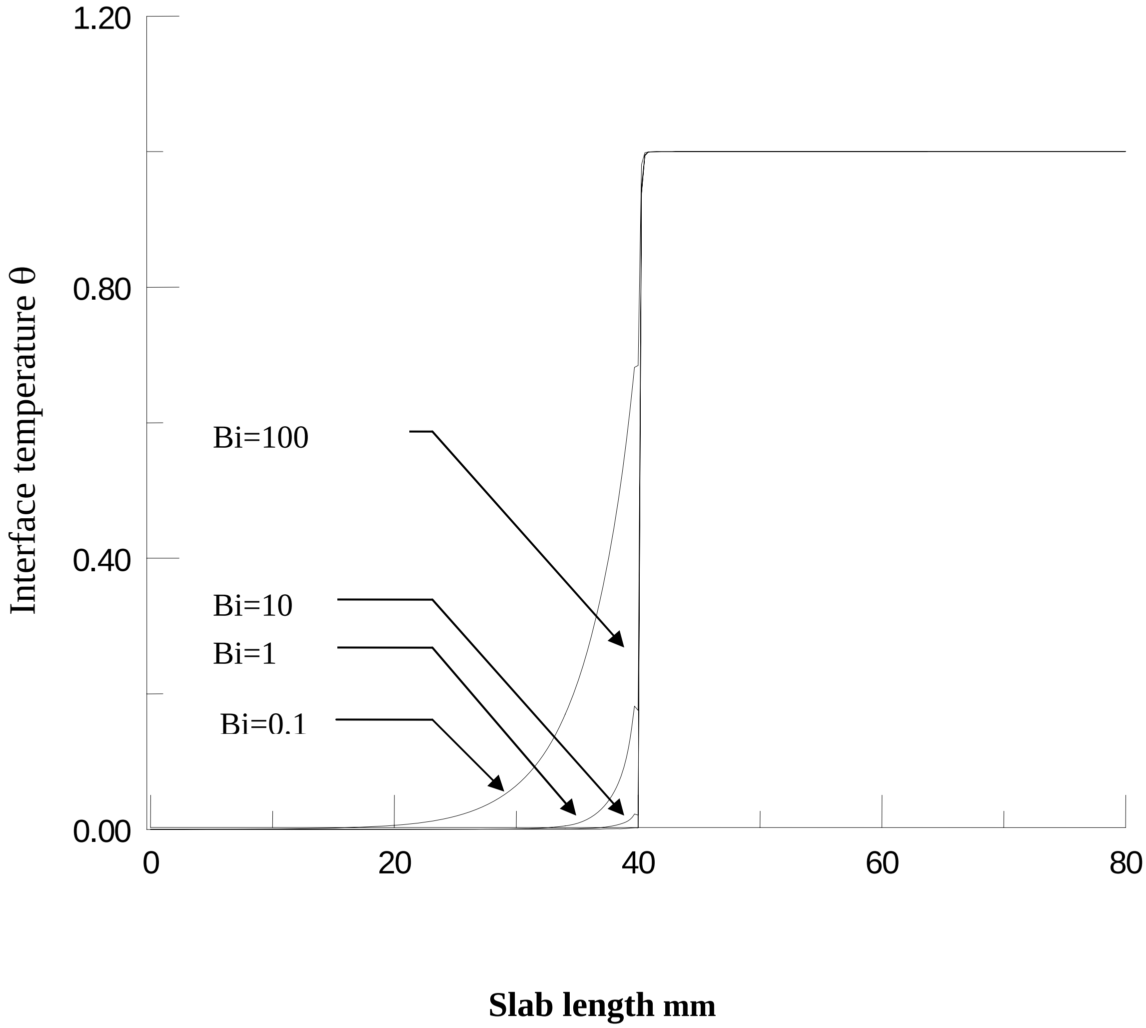
[Fig. 4.10]Temperature Contours in the slab (a) Bi=0.1 (b) Bi=1 (c) Bi=10(d)Bi=100

For a given metal in example



[Fig. 4.11] **Temperature profiles on the coolant side of slab at various Biot numbers**

For a given metal in example



[Fig. 4.12] **Temperature profiles on the interface of the slab at various Biot numbers**

For Iron Metal

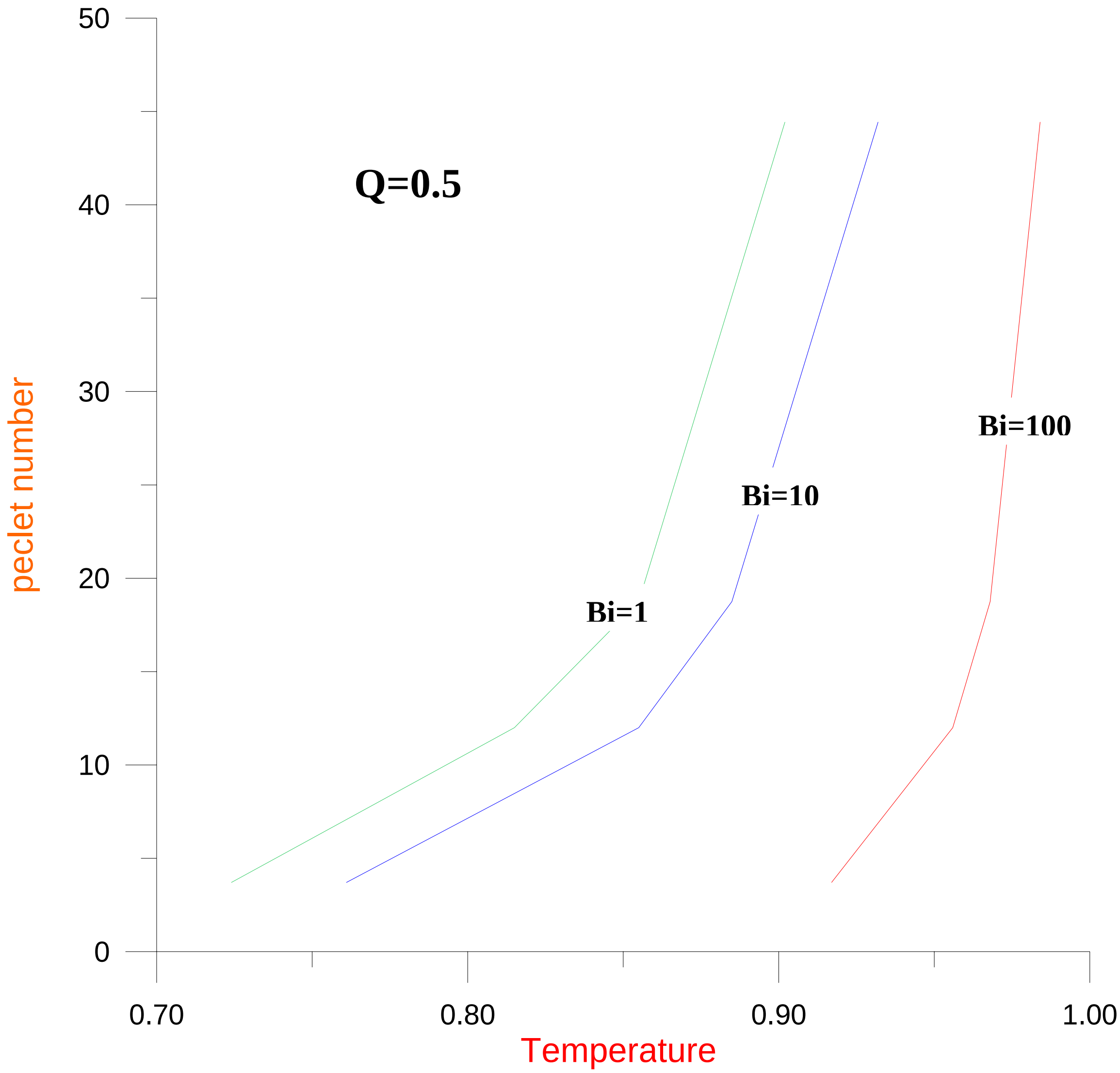


Fig 4.13 **Temperature profiles on slab at various Biot numbers and various Peclet number**

Chapter-5

Conclusions and Suggestions for Future Work

Chapter 5

Conclusions and Suggestions for Future Work

5.1 Introduction

A numerical investigation of moving boundary on continuous casting slab has been successfully carried out to analyze the complex heat transfer phenomena involved in the process. General conclusions that stemmed from this analysis are presented here, together with a brief recapitulation of some of the important remarks made earlier.

5.2 Conclusions

A numerical code has been developed to solve the conduction equation, with certain boundary conditions, for Continuous casting .The contours of temperature profiles along with the variation of Biot number are plotted. It can be concluded that the temperature is more dense near the interface. From the temperature profile of interface it concludes that the temperature gradient increases with increases in Biot number .From the temperature profile of coolant side it concludes that the temperature gradient increases with decreases in Biot number. From the temperature profile on slab it concludes that the temperature increases with increase in peclet number.

A finite difference solution for conduction controlled continuous casting of an infinite slab. The infinte physical domain can be transformed to a finite space so that the boundary conditions can be implemented in difference equations. In general Interface temperature is found to increase with increase in Biot number .the variation of interface temperature with length of slab.

5.3 Suggestions for Further Work

The present finite difference solution is believed to be capable of solving the class of conduction controlled continuous casting problems and may be extended to more involved problems, such as heat tubes and heat pipes. The suggested numerical method may be beneficial in solving non-linear continuous casting equation, attributed to temperature dependent thermo physical properties of the metal. The finite difference method may also be extended to conjugate heat transfer continuous casting model to analyze the heat transfer in solid and liquid regions separately.

Appendix

```
c***************************************************c
c        Program FDM-CONTINUOUS CASTING.FOR          c
*****************************************************
         include 'fdmslab.h'
         data b1v/0.1d00,1.0d00,10.0d00,100.0d00/
c
         open(11,file='fdmslab.out')
         open(12,file='slprof.dat')
         open(13,file='slcont.txt')
         open(14,file='interface.dat')
         open(21,file='fdmslab.in')
         read(21,*)rlength,bi,pe1,pe2,alpha1,alpha2,cond1,cond2,
        1        qsource,eps,tol,tau
         close(21)
         uwfm=0.0d00
         betam=0.0d00
         mwf=(m+1)/2
         gamma=cond2/cond1
         pe=(2.0*pe1*pe2)/(pe1+pe2)
         alpha=(2.0*alpha1*alpha2)/(alpha1+alpha2)
c
         write(*,5)m,n
         write(11,5)m,n
 5       format(/2x,'m=',i4,2x,'n=',i4)
         write(*,10)eps,tol
         write(11,10)eps,tol
 10      format(/2x,'eps=',e13.5,2x,'tol=',f13.5)
         write(*,15)pe1,pe2
         write(11,15)pe1,pe2
         write(12,15)pe1,pe2
         write(13,15)pe1,pe2
         write(14,15)pe1,pe2
 15      format(/2x,'pe1=',f8.3,5x,'pe2=',f8.3)
c
         write(*,20)alpha1,alpha2
         write(11,20)alpha1,alpha2
         write(12,20)alpha1,alpha2
```

```
      write(13,20)alpha1,alpha2
      write(14,20)alpha1,alpha2
 20   format(/2x,'alpha1=',f8.3,5x,'alpha2=',f8.3)
c
      write(*,25)cond1,cond2,qsource
      write(11,25)cond1,cond2,qsource
      write(12,25)cond1,cond2,qsource
      write(13,25)cond1,cond2,qsource
      write(14,25)cond1,cond2,qsource
 25   format(/2x,'cond1=',f8.3,5x,'cond2=',f8.3,5x,'qsource=',f8.3)
c
      write(*,60)gamma
      write(11,60)gamma
      write(12,60)gamma
      write(13,60)gamma
      write(14,60)gamma
 60   format(/2x,'gamma=',f8.3)

      pause
c
      do 30 j=1,n
      do 30 i=1,m
      u(i,j)=0.0d00
      u0(i,j)=0.0d00
      s12(i,j)=0.0d00
 30   continue
c
      do 40 j=1,n
      u(m,j)=1.0d00
      u0(m,j)=1.0d00
 40   continue
c
      do 50 i=2,m-1
      do 50 j=1,n
 50   u(i,j)=float(i-1)/float(m-1)
c
      icount=0
c     do 200 b1j=1,4
c     bi=b1v(b1j)
      iterc=0
      call grid
      call func
c
      write(*,92)bi
      write(11,92)bi
      write(12,92)bi
```

```
      write(13,92)bi
      write(14,92)bi
 92   format(2x,'bi=',f10.5)
      do 110 jj=1,n
      write(14,105)jj,y(jj),u(mwf,jj)
      write(*,105)jj,y(jj),u(mwf,jj)
 105  format(2x,i5,2x,f10.4,2x, f16.8)
 110  continue
c     pause
c     calculate mean interface temperature
c
      rint=0.0d00
      do 120 j=1,n-1
      sum=0.5*(u(mwf,j)+u(mwf,j+1))*(y(j+1)-y(j))
      rint=rint+sum
 120  continue
      umean=rint
      write(*,122)pe,umean
      write(11,122)pe,umean
 122  format(2x,'pe=',f10.5,5x,'mean intface temp=',f10.5)
c
      do 130 ii=mwf-20,mwf+20
      do 130 jj=1,n
      write(13,125)x(ii),y(jj),u(ii,jj)
 125  format(2(2x,f8.4),2x,f16.8)
 130  continue
c
      do 150 ii=1,m
      write(12,140)x(ii),u(ii,1)
 140  format(2x,f10.4,2x, f16.8)
 150  continue

      pause
c 200 continue
c
      close(11)
      close(12)
      close(13)
      stop
      end
c****************************************
      subroutine coeff
c****************************************
      include 'fdmslab.h'
c
      do 50 i=2,mwf-1
```

```
      h1=x(i+1)-x(i)
      h3=x(i)-x(i-1)
      do 50 j=1,n
      if(j.eq.1)then
      h2=y(j+1)-y(j)
      a1(i,j)=1.0/(h2*h2)
      a2(i,j)=(1.0/(h1+h3))*(1/h1+0.5*pe1)
      a3(i,j)=0.0
      a4(i,j)=1.0/(h3*(h1+h3))
      a0(i,j)=a1(i,j)+a2(i,j)+a3(i,j)+a4(i,j)+bi/h2
      else if(j.eq.n)then
      h4=y(j)-y(j-1)
      a1(i,j)=0.0
      a2(i,j)=(1.0/(h1+h3))*(1/h1+0.5*pe1)
      a3(i,j)=1.0/(h4*h4)
      a4(i,j)=1.0/(h3*(h1+h3))
      a0(i,j)=a1(i,j)+a2(i,j)+a3(i,j)+a4(i,j)
      else
      h2=y(j+1)-y(j)
      h4=y(j)-y(j-1)
      a1(i,j)=1.0/(h2*(h2+h4))
      a2(i,j)=(1.0/(h1+h3))*(1/h1+0.5*pe1)
      a3(i,j)=1.0/(h2*(h2+h4))
      a4(i,j)=1.0/(h3*(h1+h3))
      a0(i,j)=a1(i,j)+a2(i,j)+a3(i,j)+a4(i,j)
      end if
 50   continue
c
      do 100 i=mwf+1,m-1
      h1=x(i+1)-x(i)
      h3=x(i)-x(i-1)
      do 100 j=1,n
      if(j.eq.1)then
      h2=y(j+1)-y(j)
      a1(i,j)=1.0/(h2*h2)
      a2(i,j)=(1.0/(h1+h3))*(1/h1+0.5*pe2)
      a3(i,j)=0.0
      a4(i,j)=1.0/(h3*(h1+h3))
      a0(i,j)=a1(i,j)+a2(i,j)+a3(i,j)+a4(i,j)
      else if(j.eq.n)then
      h4=y(j)-y(j-1)
      a1(i,j)=0.0
      a2(i,j)=(1.0/(h1+h3))*(1/h1+0.5*pe2)
      a3(i,j)=1.0/(h4*h4)
      a4(i,j)=1.0/(h3*(h1+h3))
      a0(i,j)=a1(i,j)+a2(i,j)+a3(i,j)+a4(i,j)
```

```
        else
        h2=y(j+1)-y(j)
        h4=y(j)-y(j-1)
        a1(i,j)=1.0/(h2*(h2+h4))
        a2(i,j)=(1.0/(h1+h3))*(1/h1+0.5*pe2)
        a3(i,j)=1.0/(h2*(h2+h4))
        a4(i,j)=1.0/(h3*(h1+h3))
        a0(i,j)=a1(i,j)+a2(i,j)+a3(i,j)+a4(i,j)
        end if
 100    continue

        i=mwf
        do 150 j=1,n
        if(j.eq.1)then
        h2=y(j+1)-y(j)
        a1(i,j)=1.0/(h2*h2)
        a2(i,j)=(1.0/(h1+h3))*((1.0-gamma)/h1+pe)
        a3(i,j)=0.0
        a4(i,j)=0.0
        a0(i,j)=a1(i,j)+a2(i,j)+a3(i,j)+a4(i,j)+(2.0*bi)/(h2*(h1+h3))
        s12(i,j)=qsource/(h1+h3)
        else if(j.eq.n)then
        h4=y(j)-y(j-1)
        a1(i,j)=0.0
        a2(i,j)=(1.0/(h1+h3))*((1.0-gamma)/h1+pe)
        a3(i,j)=1.0/(h4*h4)
        a4(i,j)=0.0
        a0(i,j)=a1(i,j)+a2(i,j)+a3(i,j)+a4(i,j)
        s12(i,j)=qsource/(h1+h3)
        else
        h2=y(j+1)-y(j)
        h4=y(j)-y(j-1)
        a1(i,j)=1.0/(h2*(h2+h4))
        a2(i,j)=(1.0/(h1+h3))*((1.0-gamma)/h1+pe)
        a3(i,j)=1.0/(h2*(h2+h4))
        a4(i,j)=0.0
        a0(i,j)=a1(i,j)+a2(i,j)+a3(i,j)+a4(i,j)
        s12(i,j)=qsource/(h1+h3)
        end if
 150    continue
        return
        end
c*************************************
        subroutine func
c*************************************
        include 'fdmslab.h'
```

```
c
      iterc=iterc+1
      iter=0
      do 10 j=1,n
 10   continue
c
      do 30 j=1,n
      do 30 i=1,m
      a1(i,j)=0.0d00
      a2(i,j)=0.0d00
      a3(i,j)=0.0d00
      a4(i,j)=0.0d00
      a0(i,j)=0.0d00
      s12(i,j)=0.0d00
 30   continue
c
      call coeff
c
      do 80 j=1,n
      do 80 i=1,m
      if(u(i,j).lt.0.0d00)u(i,j)=0.0d00
      if(u(i,j).gt.1.0d00)u(i,j)=1.0d00
 80   continue
      call gsd
      return
      end
c***********************************
      subroutine grid
c***********************************
      include 'fdmslab.h'
c
      delx=1.0d00/float(m-1)
      do 10 i=1,m
      zeta(i)=delx*float(i-1)
      x(i)=(0.5*rlength)*(1.0+sinh(tau*(zeta(i)-0.5))/sinh(0.5*tau))
 10   continue
      x(1)=0.0d00
      x(m)=rlength
      x(mwf)=0.5*rlength
c     do 50 i=1,m
c     write(*,40)i,x(i)
c 40  format(/2x,i5,5x,'x(i)=',f8.3)
c 50  continue
c     pause

c
```

```
      dely=1.0/float(n-1)
      do 20 j=1,n
      y(j)=dely*float(j-1)
 20   continue
      y(1)=0.0d00
      y(n)=1.0d00

c     do 150 j=1,n
c     write(*,140)j,y(j)
c 140 format(/2x,i5,5x,'y(j)=',f8.3)
c 150 continue
c     pause

      return
      end
c*************************************
      subroutine gsd
c*************************************
      include 'fdmslab.h'
c
      iter=0
 5    iter=iter+1
      do 10 j=1,n
      do 10 i=1,m
      u0(i,j)=u(i,j)
 10   continue
      do 20 i=2,m-1
      do 20 j=1,n
      if(j.eq.1)then
      u(i,j)=(a1(i,j)*u0(i,j+1)+a2(i,j)*u0(i+1,j)+a4(i,j)*u0(i,j-1)
     2                 +s12(i,j))/a0(i,j)
      else if(j.eq.n)then
      u(i,j)=(a2(i,j)*u0(i+1,j)+a3(i,j)*u0(i,j-1)
     3                 +a4(i,j)*u0(i-1,j)+s12(i,j))/a0(i,j)
      else
      u(i,j)=(a1(i,j)*u0(i,j+1)+a2(i,j)*u0(i+1,j)+a3(i,j)*u0(i,j-1)
     4                 +a4(i,j)*u0(i-1,j)+s12(i,j))/a0(i,j)
      end if
 20   continue
c
      do 30 j=1,n
      do 30 i=1,m
      if(u(i,j).lt.0.0d00)u(i,j)=0.0d00
      if(u(i,j).gt.1.0d00)u(i,j)=1.0d00
c     write(*,*)u(i,j)
 30   continue
```

```
c
      if(iter/500*500.eq.iter)then
      write(*,40)iter,u(mwf,1)
 40   format(/2x,i6,5x,'u=',f8.5)
      endif
c
      do 50 i=2,m-1
      do 50 j=1,n
      diff=u(i,j)-u0(i,j)
      if(dabs(diff).gt.eps.and.iter.lt.maxit)goto 5
 50   continue
      return
      end
c*************************************
```

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